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Three player oligopoly model to evaluate the economic impact of cognitive radio

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ABSTRACT OF THE
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Currently, all wireless communication has been assigned to use a specific part of the spectrum. Regulation has caused the current allocation method to be very inefficient. As a solution to this problem, the concept of cognitive radio was introduced. The main idea is to make the spectrum dynamically available, so that devices can select the frequency bands that are not used at a certain time and location.

Cognitive radio still requires heavy technological development and extensive changes in regulation and the rules of spectrum use. This will also change the business ecosystem. Cognitive radio will require all the key players of the market to join the development, which in turn requires that they find the new technology profitable to them. Not only will costs and demands change, but also the whole structure of the ecosystem might undergo transformations.

In this thesis we construct a game theoretic three player oligopoly model to study how the changes brought by cognitive radio affect the business ecosystem, and whether it will be profitable for the key players to support cognitive radio. The thesis also describes a framework to help find the interactions between technological and regulatory decisions and the market parameters. With these interactions we can then estimate the value and compare different types of cognitive radio implementations.

Using parameters based on a scenario of the year 2015, the model predicts that cognitive radio *will* be profitable to all the key players. The only question is, *when* will it become reality. Our studies suggest that in the short term the shortage of spectrum might be beneficial to the network operators. We also examine possible coalitions between business sectors and basic guidelines for technological development, which all affect the possibilities of cognitive radio.

Keywords: Cognitive radio, Game theory, Oligopoly theory, Economic modeling

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Tällä hetkellä kaikki langaton viestintä on allokoitu käyttämään tiettyä osaa radiospektristä. Ajan myötä käytetty allokaatiotapa on kuitenkin osoittautunut hyvin tehottomaksi. Ratkaisuna tähän ongelmaan kehitettiin kognitiivisen radion käsite. Pääidea on tehdä spektrin allokaatiosta dynaaminen siten, että laitteet voivat vapaasti valita taajuuskaistan, joka ei ole sillä hetkellä käytössä halutussa sijainnissa.

Kognitiivinen radio vaatii edelleen runsaasti kehitystyötä ja laajoja muutoksia spektrin regulaatioon ja käyttösääntöihin. Se tulee myös muuttamaan markkinoita. Kognitiivinen radio vaatii, että kaikki markkinoiden avainpelaajat lähtevät mukaan kehitystyöhön, joka taas vaatii, että he uskovat kognitiivisen radion olevan heille tuottoisa. Muuttuvien kustannus- ja kysyntärakenteiden lisäksi, koko yritysympäristö saattaa kokea rajuja muutoksia.

Tässä diplomityössä rakennetaan peliteoreettinen kolmen pelaajan malli, jonka avulla voidaan tutkia, miten kognitiivinen radio vaikuttaa yritysympäristöön, ja onko avainpelaajien mielestä kannattavaa lähteä tukemaan sen kehitystä. Diplomityössä esitellään myös työkalu, joka helpottaa teknologian ja markkinaparametrien välisten vuorovaikutusten tunnistamista. Näiden vuorovaikutusten avulla kognitiivisen radion eri implementaatioita voidaan vertailla keskenään.

Mallin ja vuoteen 2015 perustuvien parametriestimaattien mukaan kognitiivinen radio *tulee* olemaan kannattava markkinoiden avainpelaajille. Ainoa kysymys on, *milloin* kognitiivinen radio tulee lopullisesti olemaan osa langattoman viestinnän arkea. Tulokset antavat viitteitä siitä, että lyhyellä aikavälillä spektrin loppuminen saattaa olla eduksi verkko-operaattoreille. Tutkimme myös markkinasektoreiden välisten koalitioiden mahdollisuutta sekä tuotekehityksen yleisiä suuntaviivoja, jotka molemmat vaikuttavat kognitiivisen radion tulevaisuuteen.

Avainsanat: Kognitiivinen radio, Peliteoria, Oligopoliteoria, Taloudellinen mallinnus

Preface

This thesis has been written in the Systems Analysis Laboratory of the Aalto University, School of Science and Technology, in close collaboration with Nokia Reseach Center.

I would like to thank my instructor Kimmo Berg for his help and guidance in writing this thesis, as well as his ideas on building the model. My gratitude also goes to my supervisor, Professor Harri Ehtamo, for introducing me to the field and giving me a wider perspective of this world.

I would also like to thank the Nokia Research Center representatives, my second instructor Mikko Uusitalo and Carl Wijting for providing insight on cognitive radio. Without their input, this thesis would not have been possible.

Especially, I would to like thank my friends, brothers and parents for their constructive comments that brought me here today.

Otaniemi, 2.11.2010

Arttu Klemettilä

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Chapter 1

Introduction

Mobile telecommunication is one of the fastest growing technologies of the last decade. Now, everyone not only has a mobile phone to talk and send text messages with, but to browse the world wide web, buy music, connect with friends on social networks and use numerous other services. Traditional landlines and even DSL-lines are slowly being augmented or replaced by wireless connections, as the users become more and more mobile.

All wireless communication uses the electromagnetic spectrum as a medium for transferring data. Before recent developments, the problems of wireless communication were mostly about the speed and range of the connection. Nowadays, networks are relatively fast and the connections are stable, but we face a totally new problem: the spectrum is running out, and there is not enough capacity to serve everyone.

Only a certain part of the spectrum is actually usable for communication. That part is called the *radio spectrum*. The radio spectrum has been allocated for different types of use, and is controlled by licenses and protocols that specify which user is allowed to use which part of the spectrum and how.

History has led us to a situation, where the whole radio spectrum has been divided into sections, which are then sold, rented or given to different users. The most important users of the spectrum are, besides mobile communication, TV and radio broadcasters, military radars and communication, and aeronautical and maritime navigation and communication. Even though mobile telecommunication has its own bands, there is not enough spectrum to meet the ever growing demand. On the other hand, in the many parts of the spectrum there is still plenty of space. Another problem arises when for example two or more TV stations are broadcasting in the same area. Each one requires a certain bandwidth that none of the others can use, since otherwise there will be too much interference. This does not mean the spectrum has reached its limit. On the contrary, other smaller devices could utilize the spectrum between the TV stations, without interfering the broadcast.

Cognitive radio was introduced by Joseph Mitola III [28, 29] as a solution to this problem of inefficient spectrum use. The main idea is that devices that use the electromagnetic spectrum are not tied to a single frequency as they are now. Instead, cognitive devices can sense their surroundings and select appropriate bands dynamically so that the spectrum is used more evenly. It is not just reallocation, but bands allocated to a specific user can now be used when the primary user is not using the band himself. When the original user returns, cognitive devices can find another free band. Cognitive radio also enables coexistence, so that different users can utilize the same spectrum without interfering each other, making the spectrum use even more efficient.

Cognitive radio does not only mean more spectrum to the devices that need it, but also better quality spectrum. Devices can choose to use different bands to select the properties of the connection they require at the moment. Cognitive radio is not to be confused with more efficient radio technologies, such as LTE, that improve the connection between base stations and devices. The aim is to make the whole spectrum more efficient, independent of the technology using the spectrum.

Since Mitola's first introduction of the concept, cognitive radio has been studied in various ways. There are still many open questions related to the actual implementation of the technology. The technology has to be developed, so that devices can operate in a cognitive network. Wireless devices require special rules and standards, according to which they operate. Especially with cognitive radio, co-operation between devices is needed to enable dynamic access to different parts of the spectrum.

In addition to the technological questions, cognitive radio requires changes in regulation. The global rules for spectrum allocation are set by the International Telecommunication Union (ITU), an agency of the United Nations. In addition to ITU, each country has their own national regulators. Since the spectrum is already allocated, none of it is yet reserved for the new cognitive radios. The first challenge is to find a band for cognitive radios to operate. The candidate for an initial cognitive radio spectrum band is the TV whitespaces, former television broadcast frequencies, that were freed due to the introduction of the more efficient digital television. At first, cognitive radios will operate as secondary users, that can use use the spectrum when the TV broadcaster does not.

Ultimately, the goal is to have all spectrum available for cognitive radio, but such changes cannot simply happen over night. Currently most of the telecommunication and broadcast spectrum is sold as licensed bands for typically 10 year periods. Changes are slow and the license holders do not want to give up their ownership. New spectrum licensing models are needed to enable the coexistence of the old users and new cognitive devices. Also, even if there is spectrum available for cognitive radios, who has the right to use it? Is the spectrum freely available to everyone? It has been also suggested that users could buy spectrum in real-time auctions or some other means. What is the fair way to divide the spectrum between the cognitive devices? All these questions need to be answered.

The main driving force of these changes is the companies involved in the telecommunications market. The technology must be developed by the device manufacturers and the network operators must accept the new licensing mechanisms. All this requires time and effort, which they are ready to spend, if they will get some benefit from it. Whether it is beneficial depends not only on the costs of the development of the new technology and licensing methods, but also on how the real end-users perceive the new technology.

The aim of this thesis is to find a game theoretic model to determine, under which conditions the key players in the business ecosystem find cognitive radio worth investing in. The main focus is in the market players and end users, not the technology or regulation, although they must be taken into consideration while evaluating the parameters of the model.

We will use the model to analyze some properties of the ecosystem. We will study the effect of the limitation of spectrum on the market equilibrium, as limited spectrum is one of the main reasons that cognitive radio was first developed for. Cognitive radio might also change the structure of the market by bringing new players or new pricing and marketing methods, which is why these aspects must be considered in the construction of the model. We will examine the effect of coalitions between market sectors. Since cognitive radio is still under development, it is impossible to predict what the market will actually be like. This thesis will give some guidelines, which deserve special focus in the development of cognitive radio.

The structure of this thesis is the following: In the second chapter we will examine the history and basic problems of cognitive radio. The third and fourth chapters introduce the basic methods and tools in constructing the model. In chapter five we construct the model and calculate the equilibrium solutions. Finally, chapter six applies the model to an estimated scenario of the United Kingdom in the year 2015. With this scenario we will validate the model and study some of its basic properties.

Chapter 2

Cognitive radio

In this chapter we introduce the basic principles of cognitive radio and how it could be implemented. We also discuss the current situation of cognitive radio and its main challenges. In the last section we look at the key players required in the cognitive radio business ecosystem and the principles of techno-economic modeling.

2.1 Why do we need a cognitive radio?

In a study by FCC, the Federal Communications Commission [1, 18], the current usage of the electromagnetic spectrum is very inefficient. The spectrum is assigned to license holders, such as mobile telecommunication operators, TV and radio broadcasting companies, radars and other military communication, emergency communication, GPS and so on. Some of these users utilize their section of the spectrum only for a few hours a day or even less. On the other hand, some bands, like the mobile telecommunications band, are in very intensive use. In large cities, this is already starting to become a problem.

Recently, the mobile services have become very popular among consumers, and the amount of mobile communications has been growing rapidly [34]. At the moment, there still is enough spectrum available to meet the demand, but in 5 - 10 years the situation will be much worse. Even with other technological advancements such as LTE, the spectrum will eventually run out.

Another trend in spectrum allocation is that new technologies tend to get their spectrum from the high frequency bands. These bands are characterized by a higher data rate, but also a shorter range. After the range drops below the magnitude of a few meters, there are only a few applications. This is the reason why most of the early allocated spectrum is in the lower frequency bands. Because the spectrum was not scarce at that time, the bands were allocated in a very liberal way, leaving the spectrum with wide inefficiently used sections.

Now that the mobile telecommunications sector has grown significantly, there are many different types of applications that need to use the spectrum. Some traffic requires only small data rates, but would highly benefit from a long range, which is typical to low frequency bands. An example of this kind of usage would be peer-to-peer outdoors communication far away from the main base station network. Other users might need to achieve a high data rate, but within a small area, say an office room. These users would then benefit from the use of high frequency bands. Ideally users could switch between bands dynamically and use the band they need for each application.

2.2 The definition of cognitive radio

The concept of *cognitive radio* was first introduced by Mitola in 2001 [28, 29] as a solution to the inefficient use of spectrum. With the help of recent technology advancements, such as *Software defined radio* (SDR) and sensing technologies, it could now be possible to build a device that uses the spectrum in a dynamic and more efficient way [1, 23].

FCC defines cognitive radio as "*A radio that can change its transmitter parameters based on interaction with the environment in which it operates*" [18]. This definition is not conclusive, since it leaves many things undetermined. This definition although characterizes two main features of cognitive radio [1]:

- *Cognitive capability*: A cognitive radio must be aware of its surroundings. This requires the capability to for example sense the spectrum usage in the current time, place and frequency, without causing interference to others. Thus cognitive radios can detect unused or available spectrum.

- *Reconfigurability*: A cognitive radio must know how to adapt to the surrounding spectrum. It should be able to communicate on different frequencies and with different technologies.

The long term goal of cognitive radio is to make all spectrum available for cognitive use. Currently, the spectrum is already allocated and most spectrum license owners do not want to give up their ownership. Even though cognitive radio would be made possible, it could mean that the original owners of the spectrum would still need to have a privileged right to use the spectrum. Although, others could use the spectrum, when the license owner would not.

Mitola's original work mainly focused on the definition of a standalone cognitive radio. To enable the coexistence of license owners and cognitive radio users, the Defense Advanced Research Projects Agency, DARPA, started the NeXt Generation (xG) program [12, 13]. This program specifies two types of spectrum users: *primary* and *secondary* users. Primary users have the primary right to use the spectrum, but secondary users, with the help of sensing and cognitive capabilities, may access the spectrum when the primary user is not active or when secondary usage does not cause interference.

This type of cognitive radio will have many benefits. The spectrum would be utilized more efficiently so that not only the users would get a better quality of service, but the growing demand from spectrum would by satisfied. Users could also use different bands for different needs giving them a better personalized experience. The more customizable spectrum would then arouse new services and applications that utilize the new technological possibilities. Some of these possibilities and new business models are discussed in [2, 7, 8, 17, 26].

In conclusion, it is very difficult to predict all the potential of a new technology, but it is clear that cognitive radio will benefit spectrum users.

2.3 Will there be cognitive radio?

The important question is: Will there be a cognitive radio? The answer is most likely yes, but instead we should actually ask *in what form* and *when*? This depends on several different factors. Technologically, cognitive radio is almost possible, but to actually enable its use, it has to be first approved by the regulator. Although much effort is already put into developing the rules for cognitive radio, the future is very unpredictable. Second, even if made possible through legislation and technology, cognitive radio must be beneficial to the end-users. If there are no users, there are no customers and no business. Third, the telecommunications market is large. If the firms cannot generate profit with the new products, they will not invest in their development. All three aspects must be considered when predicting the future of cognitive radio.

We believe that cognitive radio, with all its new technological features will be useful to then end-user, and if so, the regulation will be changed to best serve the people. But what will the reaction of the telecommunications market be? Our main aim in this thesis is to find the conditions under which cognitive radio will be possible in the sense of the market and firms operating in it.

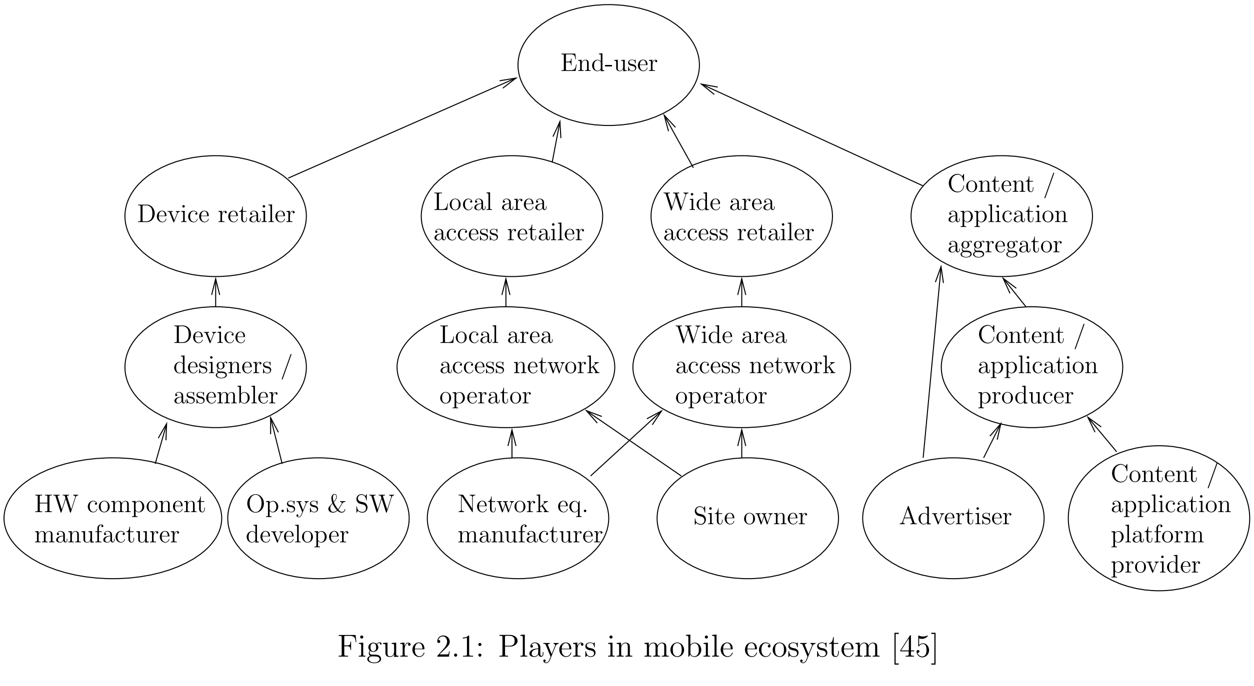


Figure 2.1: Players in mobile ecosystem [45]

The first critical step is finding the key players involved. Smura and Sorri [45] studied these players and their results can be seen in Figure 2.1. While Smura and Sorri go very much into detail, the players can be simplified to match the three main branches: device manufacturers, network access providers and content and services producers. Similar models have been introduced by Campovo and Pigneur [8] and Faber et al. [17].

Our model focuses on these three players: To use any mobile service, a user needs a device. They are manufactured by the *device manufacturers*, which sell the devices to the users. Secondly, the devices need to connect to a network or other devices using the spectrum. This connection is provided by a *network access provider* or a *network operator*, who builds the base stations and other needed infrastructure. Thirdly, after the user has a device and can connect to a network, it will need the services to use. These services are created and distributed by a large variety of *service providers*. Services range from mobile commerce to free access web portals; any service that consumers can access with a wireless device.

At least one player in each market sector must accept cognitive radio for it to happen. This does not necessarily mean that the current market players should be the same firms as they are now. For example, the network operator might be replaced by a totally new player that controls spectrum access but in a totally different way than traditional operators. Cognitive radio might even change all of the players and alter their incentives. It will also change the overall sales volumes, since in general there will be new, better quality services available. On the other hand, new technology requires investments and the marginal costs might grow due to new advanced chipsets or other technical issues. All of these changes affect the business ecosystem and therefore it will not necessarily be a good thing to all the players.

2.4 Spectrum regulation

The spectrum can be allocated in several different ways. The current three common allocation models are *licensed*, *unlicensed* and *governmental use* [5]. Cognitive radio will most likely require a new way to allocate and control the users' rights of the spectrum. So far, models such as *secondary usage*, *dynamic spectrum access* and different types of small scale *auctions* have been proposed. All these are methods are called *spectrum access models* [15].

The licensed spectrum bands are sold to the license owners, via different mechanisms [27, 46]. A license typically lasts for periods of the order of 10 years and during this period the license owner has full control of the band in the country the spectrum was bought in. The price of a band is usually very high. For example, in recent spectrum auctions, prices of 20 € to 40 € per inhabitant have been paid in the United States and the Nordic coutries. During the internet bubble in 2000 even prices as high as 620 € per inhabitant were paid in Germany [48]. All this leads to a situation, where only large telecommunications operators can afford to buy the spectrum. Also, current technologies typically require large networks of base stations, and these networks require huge investments. This leaves the end-users highly dependent of the network operator and weakens the network's adaptability to new technologies.

The government has also allocated some of the spectrum for unlicensed use. In these bands everyone is allowed to transmit data as they will, as long as they obey some certain rules. Typically, the rules limit transmit power or require a particular technology to be used. These rules are just to ensure that no one causes too much interference and that everyone has access to a sufficient amount of spectrum. Common technologies that use unlicensed bands are Wireless LAN connections, Bluetooth, small personal devices such as microphones and short range hand held transceivers.

The last type of spectrum is much like the licensed band, but the spectrum is not sold to the users. The government allocates some of the spectrum for the users to utilize as they see best. Typically, these users are the governmental organizations, such as the military and the rescue department.

What will cognitive radio do to these spectrum access models? New legislation and regulation changes are needed to enable the use of cognitive devices, because the current methods do not support them. At first cognitive radio could be built on the current unlicensed bands, but it would still require rules regarding spectrum sharing, not just the ownership of the spectrum. The most efficient method could be a totally new allocation method, designed solely for cognitive radio.

In the next sections we discuss a few proposed methods for allocating the spectrum for cognitive use. Several other methods exist and each have their advantages and disadvantages, but the ones introduced here have already gained some popularity.

2.5 Possible ways of implementing cognitive radio

The main idea of cognitive radio is to find an efficient way to allocate and share the spectrum. This definition does not restrict the way users actually interact with others or how they can use the spectrum. In order to enable the use of cognitive technologies and allow devices to coexist, special rules and agreements are needed.

The current situation also needs to be taken into account. Present spectrum owners do not want to give up their bands, and there will continue to be a lot of single band technologies in use for several years. Therefore, at least in the beginning, cognitive devices have to know how to give space to the primary users. For this purpose some mechanisms, for instance the primary database [19], have been proposed.

Even after agreeing on some mechanism to allocate a frequency and time frame to cognitive radio, users still have to allocate the spectrum between the secondary users. For this, another set of rules and agreements is needed. Several different approaches have been proposed so far, for example the Cognitive Control Radio and Cognitive Pilot Channel methods, the use of a centralized secondary database and even the use of rules similar to existing Wireless LAN connections.

Next, we will introduce some of these allocation methods. First we discuss the primary database, since it is the most probable way of controlling secondary access in the near future. Afterwards, we describe some methods for controlling secondary use.

2.5.1 Primary database

The next implementation of cognitive radio technology will most likely be the primary database. During the last few years in the US, the digitalization of television has made available some portions of the spectrum. These bands are called whitespaces, and they are being made available for cognitive use. The legislation is not yet final, but basic guidelines have already been set by the FCC [19]. The planned technology to use is called primary database. Some things are still open, but decisions will probably be made within a few years.

The main idea is that the primary users have to report whether they will use their spectrum on a certain date and in a certain area or not. This data is gathered into a database accessible by an Internet connection. Now if a secondary user wants to use the spectrum, he will first have to query the database and check if there is free spectrum available at his location for the current day. Queries are valid for one day. If the primary user is not using the spectrum, the secondary user may use it. As a safety mechanism, the secondary users must listen to a spectrum channel before using it, to make sure the primary user is not using the spectrum, even though the database reports differently.

As said, some issues are still open. For example, who will be the operator of the database? It could be a governmental organization or a private company. In the case of a private company, one or several of the current network operators could start operating it. It could also be a totally new company aimed directly for this purpose. So far, at least the advertisement based search provider Google and six other companies have announced their interest to operate such a database [20].

The organizer of the database has a large influence on how the whole system works. Will the database queries be free, will there be a per day fee or monthly subscription, or will everything be based on advertisements?

2.5.2 Spectrum allocation between secondary users

The primary database as described before does not define how the secondary users should divide the available spectrum. These methods can be roughly divided into three groups: distributed, centralized and open access. In distributed methods the cognitive devices agree on the allocation with each other, based mostly on sensing data obtained by all the nodes. Centralized methods have a centralized database, similar to the primary database, which controls all allocation. In open access methods, the spectrum is not controlled in any way, except for a small set of rules, mainly to limit transmit power and a few other properties.

One possible answer to this is the distributed Cognitive Control Radio (CCR), introduced in [16]. CCR as such does not take into account the primary users, but it was developed to allocate spectrum to a group of cognitive radios. Devices communicate via a special control channel. On this channel, devices organize into small groups, called Cognitive Radio Networks (CRN) that operate a certain frequency band. The CRNs scan the surrounding spectrum and together divide it using a specific set of rules. Each CRN joins the central awareness network, or CCR network, via one or a few nodes who agree on spectrum usage at the whole CCR level. See Figure 2.2 for an illustration the network structure.

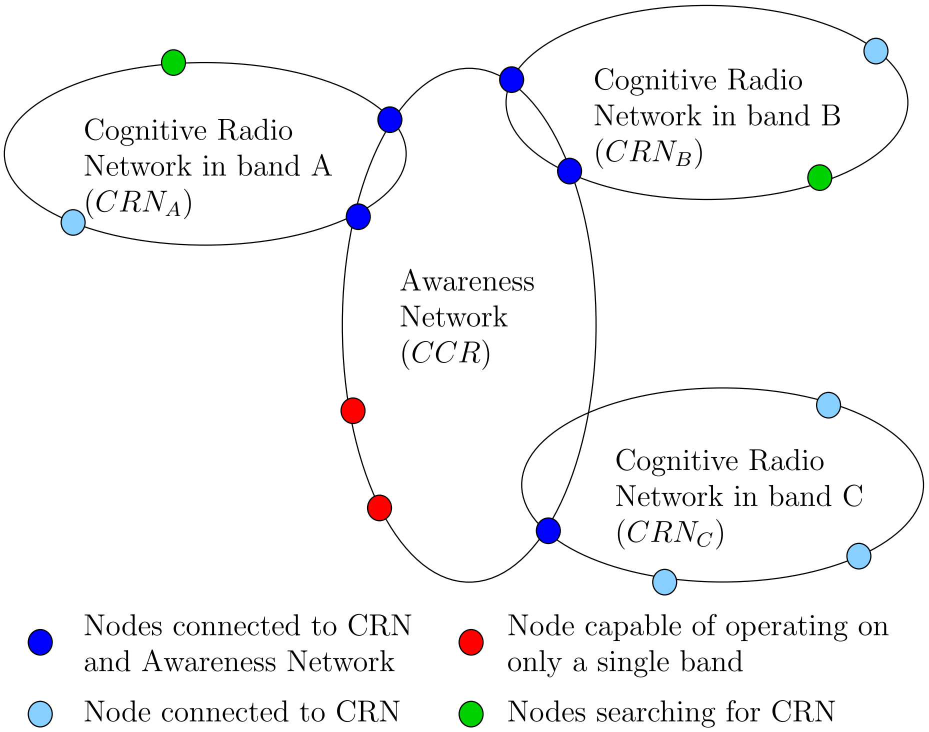


Figure 2.2: Structure of a Cognitive Control Network [16]

An alternative to CCR is the Cognitive Pilot Channel (CPC) [3, 10, 14]. As CCR, the CPC specifies a certain easy detectable control channel on which spectrum allocation messages are transferred. It has not yet been specified, which instance owns and operates the pilot channel, but the network operator, a third party, or some hierarchical combination of these have been suggested.

Distributed methods in general require more control channel communication than the centralized database solutions, but they also provide a better way of allocating the spectrum. Because spectrum utilization estimates are based on local, real sensing information and not computed values of the database, distributed methods can achieve a more efficient allocation. A distributed network is also more agile and dynamic than a central database.

Distributed methods like CCR have also received some criticism, since they require more from the devices. Sensing and communication require more battery than a simple database query. Distributed methods also need a way to agree on priority. What will happen, if there is spectrum available to only one node, but several nodes want to use it?

The field for distributed spectrum allocation is still very open, and even the simpler centralized database implementations are awaiting their first realization. Although, it is highly probable that at some point at least some bands will enable a distributed spectrum allocation method, since when designed properly, it provides a more efficient allocation.

2.5.3 Mesh networks

One possible topology that cognitive radio could advance are mesh networks. Mesh networks consist of nodes that have a direct node-to-node link. This link is very short compared to traditional node-to-base station links. Messages can also be relayed to other nodes. In practice this results in a local area wireless network. An example is illustrated in Figure 2.3.

This type of cognitive networks have been proposed, for instance one of the most prominent ones is called CogMesh [9]. Similar, non cognitive networks already exist, but they have not had a large commercial impact. Currently, these networks are built using unlicensed spectrum, but with cognitive radio these networks would be more dynamic. Networks could select whether they would want to prefer a longer range or higher data rate. In distant areas they could make full use of the spectrum in the case that the spectrum is only partially utilized, or the base station network is not sufficient.

The introduction of mesh networks to the large audiences could make free short distance calls possible. It could also give birth to new types of services and applications, that make use of the new capabilities.

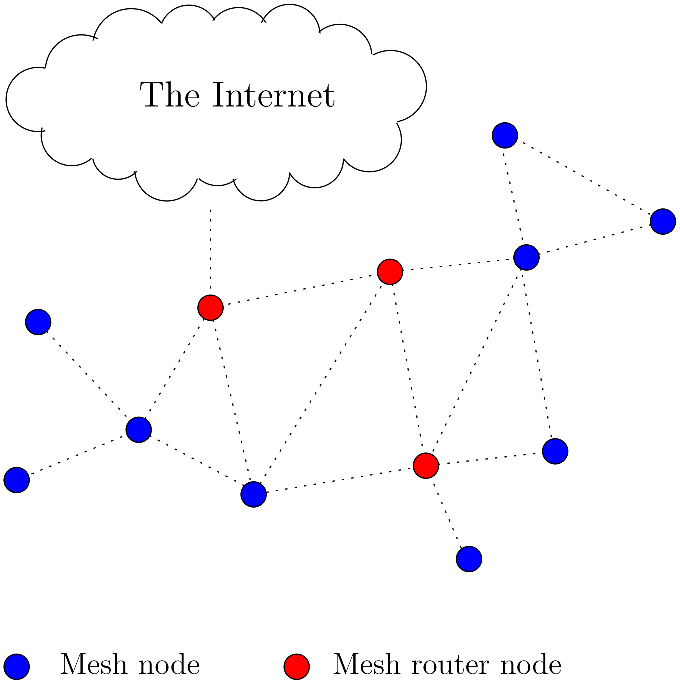


Figure 2.3: Mesh network

Mesh networking over cognitive networks could be possible without traditional network operators. This is because mesh networks do not need a centralized network to operate. This means that cognitive radio could have a future, even without the support from operators. Of course, this would limit the scale of the technology.

2.6 Value chains and value networks

Traditionally, business ecosystems have been modeled with the *value chain* concept. Value chains are simple linear models, that start with the basic raw material and ends with the end user. Along the chain there are firms that buy products from the previous links and sell them with some added value to the next link. In the chain, the commodities move forward and money moves backwards.

Value chains are relatively easy to analyze. Each link adds value to the product, which is then paid for by the next node. All profit is originated from the end user and flows to the firms along the chain.

Recent economic studies have pointed out, that this kind of value chain thinking might not be sufficient to model modern business ecosystems. This is especially visible with the modern telecommunications business [25, 35, 41, 47]. It is very large and complex, and there is a lot more to it, than just users buying devices, subscriptions and services from the providers. In the US, phones and subscriptions are often sold in bundles, in which you get the device almost for free and pay only for the subscription. Mobile phone companies like Nokia are also entering the service sector with their own service portals and forming collaborations with other service providers like Yahoo!. This leads to a situation that in the end, it is very hard for the customers to know what they are actually paying for.

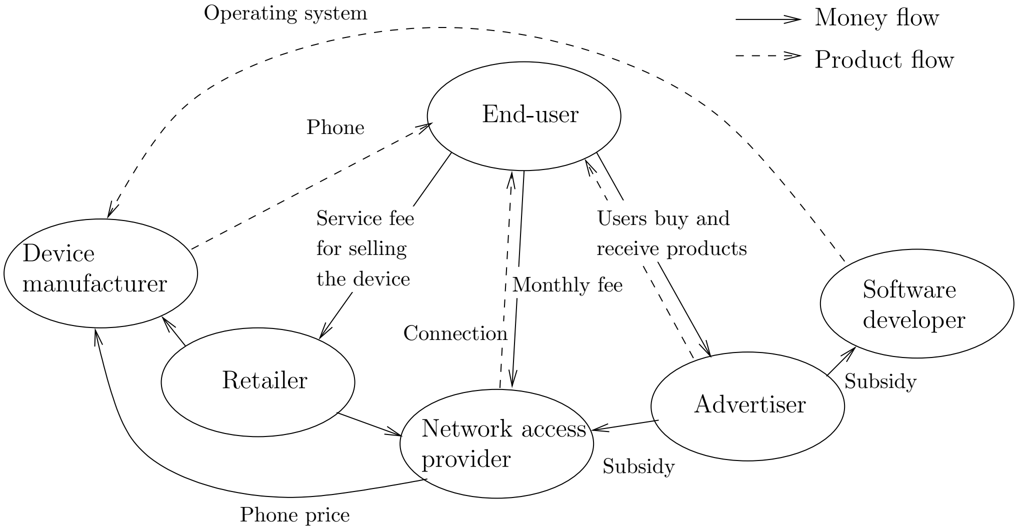


Figure 2.4: Example of a value network

To help model these situations, the concept of a *value network* has been introduced [37]. In a value network the different links form a network that interacts with each other in various ways. Unlike a value chain, money from the end customer can flow to the provider through a very complex route, and the commodities travel a totally different route.

As an example, consider a national network operator selling a smart phone in a bundle with a flat fee data plan. The value network is illustrated in Figure 2.4. The customer buys the bundle and gets the phone. Now, even though the bundle has a fixed monthly fee, some of the fee goes to the network operator and some to the phone manufacturer. The operator might also pay something to the device manufacturer, in order to get privileged rights to sell a certain model. The phone uses a free operating system, which is partially funded by advertisements. The advertiser also pays subsidies to the network operator to send weekly SMS-messages to subscribers.

As one can imagine, the analysis of such a network is significantly more difficult than a simple value chain. Sometimes it is possible to simplify the network in a way that we leave out the complex loops and cross relations and narrow the whole network down to a few main components. The inner structure of the component can be rather diverse, but from an outside point-of-view, the whole component is just one link in a smaller value chain or network.

In our example scenario, we could simplify the chain into three players: the end-user, the network operator and the device manufacturer. Users will pay a monthly fee to the network operator in exchange for service. The network operator then pays the device manufacturer for the devices. Advertisements could be considered as part of the network sales. Although now after determining the profits and sales of each player, it should be noted that although the middle player is called the network operator, other players get a share of its profits.

There are also other difficulties related to predicting the future. Since not a single cognitive radio device has been manufactured, it is very hard to estimate, what the reaction of the consumers will be. It depends highly on the actual properties of the devices and what new services are made possible.

Chapter 3

Game theory

In this thesis, we will build a game theoretic model to study the market equilibrium of the mobile business ecosystem. First, we will need some basic definitions, such as what is a *game* and what is the *equilibrium solution* of that game. Later we will construct the model with the help of these tools.

Game theory studies situations where two or more *players* have the option to select their actions from a set of alternatives. The basic principles of game theory were laid down by Nash [30, 32]. He focused on the research of non-cooperative games, in which the players make their decisions independently and base their decisions on rationality.

3.1 Basic definitions

A *game* consists of a set of players $\mathbb{P}$ and their possible *actions*. We denote the amount of players by N and the set of possible actions for player i with $\mathbb{S}_i$. In addition to this, we define the player's *payoff* function u_i, which is a mapping from the player's actions $\mathbb{S}_i$ to a player's utility or value. We denote

$$u_i : \mathbb{S}_1 \times \mathbb{S}_2 \times ... \times \mathbb{S}_N \longmapsto \mathbb{R} \tag{3.1}$$

as the payoff function for player i. Now, we can define a game G as

$$G = (\mathbb{P}, \mathbb{S}_1, ..., \mathbb{S}_N, u_1, ..., u_N). \tag{3.2}$$

In the beginning of the game, each player chooses one action from his set of available actions. After this, the utility for each player is given by the players' payoff functions. Players are usually assumed to be *rational*, which means that they will pick the strategy that gives them the best payoff. The main aim of game theory is to find different *equilibrium solutions* to these games. Equilibrium solutions define which actions should each player choose so that no one would want to change his action.

As an example, let us consider two mobile network operators. Both firms have to set a price for a monthly data plan. To simplify the situation, each firm can choose either a high (H) price or a low (L) price. If they both select low price levels, they sell a lot of data plans but with low margins, resulting in the profit of 1 billion euros. If both select the high price, they sell a bit less, but with higher profit, generating 5 billion euros of profit. If one firm sells at a low price and the other with a high price, most customers choose the low price data plan. This means that the low price firm will make 7 billion euros, while the other will get nothing.

Now, the possible actions for both player are $\mathbb{S}_i = \{\mathrm{H}, \mathrm{L}\}$. The game can be represented as a matrix:

		Operator B	
		H	L
Operator A	H	5,5	0,7
	L	7,0	1,1

Here the rows represent the strategies for player A and the column strategies for player B respectively. In each cell of the matrix, the left number indicates player A's profit, if that combination of strategies is played. Respectively, the right number indicates the profits of player B.

Unlike in the example, strategies need not to be discrete variables. They can also be continuous or any other elements of an arbitrary set. For instance, in the previous example, we could agree that the firms could select any price, not just one of the two levels.

3.2 Nash equilibrium

We assume that the players are rational, which means that they will always choose the option that gives them the most utility. If we assume that all other players have chosen their strategy, then the players *best response* is the strategy that gives him the best utility in respect to the other players' choices. In other words, the Nash equilibrium is a set of strategies, that no player wants do deviate from alone.

The Nash equilibrium is the point, where these best responses intersect. This means that if strategies S_1^*, S_2^*, ... S_N^* form a Nash equilibrium if

$$S_i^* \in \underset{S_i \in \mathbb{S}_i}{\arg\max}\; u_i(S_1^*, S_2^*, ..., S_i, ...S_N^*) \quad \forall i \in \{1, ..., N\}. \tag{3.3}$$

In the case of our example, the strategy pair (H,H) is not a Nash equilibrium, since it is more beneficial for either one to change his strategy to L. The strategies with different prices, i.e. (H,L) and (L,H), are not equilibria either, since the player selecting the high price would benefit from changing to the low price. (L,L) on the other hand is a Nash equilibrium, since neither player will want to change his strategy, unless the other one does so, too.

The described game is an example of a two player game in which the equilibrium is not Pareto optimal, or in other words, optimal in the sense of total utility. It would be better for both to agree that they would sell at the high price and then collect the total 10 billion euros of profits.

3.3 Alternative equilibria

The Nash equilibrium is a state that none of the players want to deviate from, if we already are in the equilibrium. The setting also assumes the players to be equal in the sense that no one can change their choice after the other one has already made up his mind.

But what if one of the players can select his strategy first, and then the others can only react to this. The first player can assume that the others are rational and therefore can predict the responses of the other players. Based on this prediction he can select his own strategy so that he will get the best utility.

This type of game is called the *Stackelberg game*. The players are ordered and starting from the first one, players choose their strategies. Based on that selection, the next player chooses his strategy and so on.

In some situations and models, this type of equilibrium is sometimes more realistic than the Nash equilibrium. In the case of a market where players set prices for products, on the long run the Nash equilibrium might be closer to the actual real world prices. This is because different players have time to react and change their prices. Small changes are possible, but in general, prices tend to drift towards the Nash equilibrium.

On the other hand, short term prices will most likely follow different rules. If one players decides to quickly change his price, the others cannot react to that price immediately because of supply contracts and other practical reasons. This happens also, if the market has one clear leader. The market leader can, with the help of its market share, change the prices more than its smaller competitors. If a smaller company would change its price a lot, customers would move to another company. Large firms tend to have better customer relations and the loss of customers is not as probable.

In the telecommunications market, this type of multi stage game is also clearly visible. People will first buy a device, and do not necessarily think much about how they will use it. After this they will start looking for a network data plan, and finally, start using the device to enable different types of services. This could indicate a clear three-phase game. On the other hand, the market is very competitive and on the long scale the products are replaced often, which leads to a relatively continuous market.

3.4 Cooperative games

In non-cooperative N-player games, the players usually maximize their utility independently. The solution of the game is some equilibrium point and the utilities are realized according to the respective actions. In some situations, it could be better for the two or more of the players to form a coalition. This means that the coalitioners decide their strategies together, and afterwards possibly share the utility. This type of game is called a *cooperative game*. If forming the coalition does not cause any new costs, the coalitioners will get at least as much as without the coalition, since the players can choose the same strategies they chose without the coalition.

Utilities are said to be *transferable*, if you can give some of your utility to another player. This is usually possible in the case where the utility is measured in the amount of some countable commodity, money for instance. In a coalition, if the utilities are transferable, the coalitioners can just maximize their total utility and then divide the utility in some fair way. Even if the utilities are not transferable, the coalition can still achieve more utility than the coalitioners would gain without joining the coalition.

The basic prerequisite for the coalition to be possible is that all players get as least as much utility they would get without the coalition. After this prerequisite has been fulfilled, the players have to find some some way to distribute the gained value. In the case of transferable utilities this can be done more freely, but even then it is not clear how it should be done.

The two most common ways to split the utility are the Nash [31] and Kalai-Smorodinski [24] -solutions. In both methods utility is measured against the worst case scenario, what is the worst possible split so that the coalition will still happen. The Nash solution is the point where both players get an equal amount of extra utility compared to the worst case point. The Kalai-Smorodinski solution on the other hand is the point where the ratio of extra utility compared to the worst case solution is the same as the ratio of the dream point. The dream point is an unachievable point, where both players get the highest possible utility.

3.4.1 Shapley value

One way to split the extra utility between the coalitioners is the *Shapley value*. It is a number that tells how much each coalitioner should get from the utility being split and it is based on calculating how much each player brings to the coalition.

To calculate the Shapley value, we need to know the total utility provided by each possible coalition. Let us denote that value with the function $V : S \mapsto \mathbb{R}$, where S is a subset of the set of players $\mathbb{N}$.

Next, to study the coalition $\mathbb{A} \subset \mathbb{N}$, we take all possible subsets of $\mathbb{A}$, which include a certain player i. Next we calculate how much additional value this player i brings to each of these subcoalitions by subtracting

$$V(S) - V(S \setminus \{i\}), \quad S \subset \mathbb{A}, i \in S. \tag{3.4}$$

Next we calculate the average of these differences which yields the total value of player i in the coalition $\mathbb{A}$:

$$\phi_i(\mathbb{A}) = \frac{1}{|N|!} \sum_{S \subset \mathbb{A}, i \in S} \left[V(S) - V(S \setminus \{i\}) \right]. \tag{3.5}$$

This is called the Shapley value for player i. Note that the Shapley value sums up the total value of the coalition, $V(\mathbb{A})$, and thus it is a possible way to share the utility generated by the coalition.

Chapter 4

Economic theory

The cognitive radio business ecosystem consists of several firms selling their products to end-users. In order to construct a model we will have to define some economic terms, such as demand and elasticity. This then leads us to oligopoly theory, the study of markets with a few competitors.

4.1 Microeconomic theory

The basic tool of microeconomics is the demand function. It is a function that tells how many customers are willing to buy a commodity for a certain price. In addition to the absolute level of the function, a very important aspect of the demand function is its elasticity. Formally, the *elasticity of demand* is defined as

$$E(p) = \frac{dD(p)}{dp} \cdot \frac{p}{D(p)}, \tag{4.1}$$

where $D(p)$ is the demand function at price p. Elasticity can be interpreted as the ratio of the relative change in demand and the relative change in price. The elasticity describes how sensitive the demand is to price changes. The value is usually negative, since lowering the price rarely lowers the demand, if all other factors stay constant. This also implies that demand function are typically decreasing.

Typical values of elasticity range from -0.01 to -5. For example insulin has an estimated elasticity of -0.01, rice in the US -0.5 and cars approximately -3 [22, 36, 38]. When the price of a product with a high elasticity rises, the demand goes down quickly. Low elasticity products on the other hand sell about the same amount, independent of the price. Therefore necessities usually have a low elasticity, and on the other hand, commodities that are not as critical have a high elasticity.

As a simple demand function, we can examine the linear function

$$D(p) = a - bp, \tag{4.2}$$

where p is the price, a is the demand when the price is 0, and b is a constant which tells how much the demand goes down if the price rises by one unit. The constant b and the elasticity are connected, but where the elasticity indicates the relative changes in demand, b represents the absolute changes. The elasticity of a linear demand function is not the same at every price, since it represents the relative changes.

$$E(p) = \frac{\Delta D(p)}{\Delta p} \cdot \frac{p}{D(p)} = -b \cdot \frac{p}{a - bp} \tag{4.3}$$

If we assume a linear demand function and observe the demand of some commodity with elasticity E at price and demand p_0 and D_0, we can approximate the elasticity at price p^0 as

$$E = \frac{\Delta D(p)}{\Delta p} \cdot \frac{p_0}{D_0} = \frac{D(p) - D_0}{p - p_0} \cdot \frac{p_0}{D_0}. \tag{4.4}$$

We can now construct a linear demand function that has elasticity E at price p^0 by solving the demand function $D(p)$ from Equation (4.4):

$$D(p) = D_0 \left(1 - \frac{E\Delta p}{p_0}\right). \tag{4.5}$$

Even though the presented function is very simple, it will give a good approximation of the demand at prices near p_0. The function and its elasticities are illustrated in Figure 4.1.

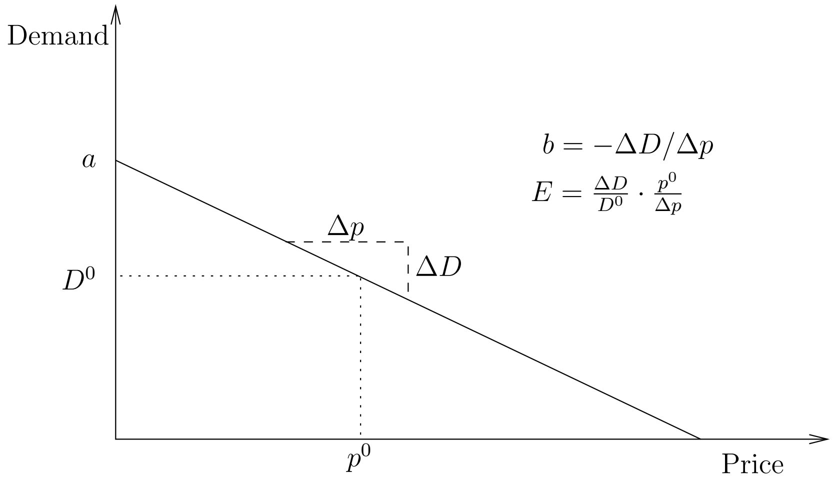


Figure 4.1: Demand function

Producing commodities to meet the demand always generates costs. They can be divided into *marginal costs* and *fixed costs*. Marginal costs are the same for each produced unit, and fixed costs are the same, independent of how many products are made. Typically marginal costs consist of resources and salaries, and fixed costs include investments and research costs.

Consider a firm producing a commodity. The goal of the firm is to produce profit, which can be calculated with the formula

$$\pi(p) = (p - c_M)D(p) - c_F, \tag{4.6}$$

where p is the price, c_M the marginal cost, c_F the fixed cost, and $D(p)$ the demand. Naturally, the firm tries to maximize the profit, usually by selecting the best price. If the price is too high, there will not be enough demand, and if the price is too low, the will not be enough margin to generate profit.

Let us examine the situation with the linear demand function from Equation (4.2). Now we can write

$$\pi(p) = (p - c_M)D(p) - c_F = (p - c_M)(a - bp) - c_F. \tag{4.7}$$

What is the optimal price, that gives the maximal profit? We can solve this using the first order optimality conditions:

$$0 = \frac{\partial \pi(p)}{\partial p} = -(p - c_M)b + (a - bp) \tag{4.8}$$

which yields

$$(p - c_M)b = a - bp \quad \Leftrightarrow \quad p = \frac{a + c_M b}{2b}. \tag{4.9}$$

As can be seen from the solution, the optimal price depends on the demand function and costs. Especially, it depends on the elasticity via the parameter b and marginal costs c_M.

4.2 Oligopoly theory

As long as there is only one firm in the market, the market is called *monopolistic*. But if there are more firms selling the same or a similar product, there will be *competition*. Depending on the amount of sellers, the market is called a *duopoly* (2) or *oligopoly* (some). If there are so many sellers, that they lose all their market power, the market has *perfect competition*, and can be considered as a market with infinite sellers. Under perfect competition, the prices are forced to the same level as the marginal costs. There are no profits, only the costs are covered. This is because there will always be someone willing to sell at a lower price, until it is not profitable.

To model competition between two players, we will introduce the *Bertrand duopoly model* [6]. Assume two firms selling commodities with prices p_1 and p_2. The basis of the model is the demand function

$$D_i(p_i, p_j) = a - b_i p_i + \gamma_i p_j, \quad i = 1, 2 \tag{4.10}$$

where b_i and γ_i are constants. As discussed before, b_i should always be positive, but γ_i can be either negative of positive depending on whether the two products are *complementary* ($\gamma < 0$) or *substitutes* ($\gamma > 0$). If $\gamma = 0$, the products are independent and form two separate markets.

Now the two firms optimize their prices simultaneously. Both have to select a price so that they get as high profit as possible, but they also have to take into account the other firm's strategy. Let us find the Nash equilibrium of this game. If player j sets his price to p_j, player i will get the profit

$$\pi_i = (p_i - c)(a - b_i p_i + \gamma_i p_j). \tag{4.11}$$

Differentiating with respect to p_i, we get as the first order conditions of the maximum

$$a - b_i p_i + \gamma_i p_j = b_i(p_i - c), \tag{4.12}$$

which yields

$$p_i = \frac{a + \gamma_i p_j + b_i c}{2b_i}. \tag{4.13}$$

If we assume a symmetric situation between players i and j, we can calculate the Nash equilibrium by solving the equation for both players simultaneously. This gives us the equilibrium of

$$p = \frac{a + bc}{2b - \gamma}. \tag{4.14}$$

From this form we see, that if the two products are independent ($\gamma = 0$), the prices are the same as in the monopolistic situation in Equation (4.9). If the products are substitutes ($\gamma > 0$), the prices will be higher than in a monopolistic market. Respectively, if the products are complements ($\gamma < 0$), prices will be lower.

The Bertrand model originally was a response to the model introduced by Cournot in 1838 [11]. In the Cournot-model the basic setting is the same, but instead of prices, players set the production quantities. These two games are dual games of each other. As Bertrand [6] claimed, the setting of the Bertrand model represents the actual market in a more realistic way, since firms compete with price, rather than quantity. More on the differences between the Bertrand and Cournot models can be read for example in [44, 49].

Chapter 5

Model for evaluating the economic impact of cognitive radio

We will now construct a model to evaluate the impact of cognitive radio on the telecommunication market. To do this, we will first need to identify the players involved in the business ecosystem and estimate their utilities and incentives. Secondly, we will construct a game theoretic oligopoly model to analyze the ecosystem and determine its Nash equilibrium.

The model will depend on several parameters, such as costs and demand levels. Since there is high uncertainty in the parameters, it is important to analyze the model's sensitivity and identify the most critical parameters. We can extract some basic properties of the model from the equilibrium's analytic solution.

We will also construct several scenarios to apply the model to. Since a single cognitive radio device has not yet existed, it is very difficult to predict, what type of market it will have. There are several options and therefore we will study different scenarios and how they portray the future of cognitive devices.

5.1 Player setting

The players of the game were discussed in Section 2.3. The original owner of the spectrum, the government, sells, gives away, or in some other way sets the spectrum available to the others. We can assume that this is already done, so the government is then left out of the game. Next, the three business sectors, device manufacturers, network operators and service providers will sell their product to the users for some price. The users on the other hand can buy the products for the given price. Depending on the price, not everyone will buy the product, since it might be too expensive. Therefore we will model the amount of users ready to buy the products with a demand function.

For simplicity, we will assume that each of the three sectors are controlled by a monopolistic firm, which has control over the whole sector. This is as such not fully realistic, since in every country there are a few large companies on each sector forming an oligopolistic market. Services typically have even more firms competing, although they represent different types of services and therefore do not all compete directly. In reality the firms do not have full control of the price, but we assume that the market forces will set the prices in a way that each member of the oligopolistic market will act similarly.

Now, we have three firms, which set a price for their commodity, and the users, who pay for them. Each firm will want to maximize their profit. In the sense of game theory, the end-users are actually not players, since they only react to the prices they are given. This ends us up with a three player game, where everyone tries to maximize their own profit subject to the demand, which is determined by the prices of all the three players.

To simplify the notation, we will denote the players, device manufacturer, network access provider and service provider with indexes $1, 2$ and 3 respectively. Let the price for each players product be p_i and their marginal cost c_i. We will leave the fixed costs out, since they do not affect the equilibrium, only the absolute level of the generated profit. With these agreements, we can calculate the players profit

$$\pi_i = (p_i - c_i)\, D_i(p_1, p_2, p_3). \tag{5.1}$$

For the demand function we will use the linear function

$$D_i(p_1, p_2, p_3) = D_i^0 \left(1 - \sum_{j=1}^{3} \frac{E_{i,j} \Delta p_i}{p_i^0} \right), \tag{5.2}$$

where $E_{i,j}$ denotes the elasticity of demand for player i for the price set by player j, D_i^0 the initial demand and p_i^0 the initial price. Δp_i is the deviation of the set price from the initial price, i.e. $\Delta p_i = p_i - p_i^0$.

5.2 Coalitions

As discussed earlier, this model only takes into account end-users buying goods and paying for them directly. It would be very difficult to model all possible advertisements and other money transfers between players. Instead we can determine what would happen if some of the players formed a coalition. This would mean they can set their prices together, while they maximize their total profit.

The interpretation of a coalition can be different in different scenarios. The simplest interpretation would of course be, that they agree on the prices together and then share the profit in some fair way. We do not discuss how the profits should be divided, only just the total profit for both of them.

Another interpretation could be, that one player pays a fee or subsidy to another player in order to make the other player set a certain price. Technically they are still different firms, but the other can control the other with these transactions. This kind of arrangement might not be as effective as a pure coalition. The effectiveness depends on the firms and many small factors that are not taken into account in this model.

The third interpretation would be that the one player owns or is in total control of the other. For instance, Nokia, a large device manufacturer, has launched their own service portal called Ovi in 2008. Now the service and device players are actually the same firm, even though they represent different players in the game.

Independent of the interpretation, coalitions are needed to analyze different types of value networks. Value networks as such are too difficult to model game theoretically because it is impossible to construct an exact network without detailed information about the market. The aim of this thesis is to find the outcome of the market at a business sector level. Each player can consist of a separate value network, we just do not consider them now.

5.3 Necessary conditions for the Nash equilibrium

In this section we will determine the Nash equilibrium of the game. As discussed in Chapter 3.2, the equilibrium is the intersection the players' response functions. First we will calculate the optimal response for each player and then solve the equilibrium.

5.3.1 Conditions for the basic situation

Each player tries to maximize its profit by changing the price of the product it is selling. This player's problem can be written in the form of

$$\max_{p_i} \pi_i = (p_i - c_i)D_i. \tag{5.3}$$

By substituting the demand from Equation (5.2),

$$D_i = D_i^0 \left(1 - \sum_{j=1}^{3} \frac{E_{i,j} \Delta p_i}{p_i^0} \right), \tag{5.4}$$

and calculating the first order necessary conditions for the optimum, we get

$$0 = \frac{\partial \pi_i}{\partial p_i} = \frac{\partial}{\partial p_i}(p_i - c_i)D_i = \frac{\partial}{\partial p_i}(p_i - c_i)D_i^0 \left(1 - \sum_{j=1}^{3} \frac{E_{i,j} \Delta p_j}{p_j^0} \right). \tag{5.5}$$

Now by differentiating, reorganizing the terms and dividing both sides with D_i^0 yields

$$1 + \sum_{j=1}^{3} E_{i,j} + \frac{E_{i,i} c_i}{p_i^0} = \sum_{j=1}^{3} \frac{E_{i,j} p_i}{p_j^0} + \frac{E_{i,i} p_i}{p_i^0}. \tag{5.6}$$

This applies for all $i = 1, 2, 3$. All the equations are linear with respect to the prices p_i, so we can write them in matrix form

$$M\hat{p} = b, \tag{5.7}$$

where the matrix M is of the form

$$M = \begin{bmatrix} 2E_{1,1} & E_{1,2} & E_{1,3} \\ E_{2,1} & 2E_{2,2} & E_{2,3} \\ E_{3,1} & E_{3,2} & 2E_{3,3} \end{bmatrix}, \tag{5.8}$$

$\hat{p}$ is a vector with the prices normed with their initial values p_i^0:

$$\hat{p} = \begin{bmatrix} \frac{p_1}{p_1^0} & \frac{p_2}{p_2^0} & \frac{p_3}{p_3^0} \end{bmatrix}^T, \tag{5.9}$$

and b is the vector

$$b = \begin{bmatrix} 1 + \sum_{j=1}^{3} E_{1,j} + \frac{E_{1,1}c_1}{p_1^0} \\ 1 + \sum_{j=1}^{3} E_{2,j} + \frac{E_{2,2}c_2}{p_2^0} \\ 1 + \sum_{j=1}^{3} E_{3,j} + \frac{E_{3,3}c_3}{p_3^0} \end{bmatrix}. \tag{5.10}$$

Now the equilibrium prices can be solved by

$$\hat{p} = M^{-1}b, \tag{5.11}$$

if M is invertible.

Because the elasticities for a player's own price are typically higher than the ones for the other players' prices, the diagonal elements of M are larger than the non-diagonal elements. According to the Gershgorin circle theorem [21], this means that the eigenvalues of M are non-zero and therefore M is invertible.

5.3.2 Conditions for two player coalitions

Let us examine the situation where two of the three players form a coalition. Let players i and j be the coalitioners, leaving player k out. The outsider still maximizes its profit against the prices set by players i and j, so the response function satisfies the same optimality condition (5.6) as in the non-coalition game. The coalitioners, on the other hand, maximize the sum of the players' profits:

$$\max_{p_i,p_j} \pi_i + \pi_j = (p_i - c_i)D_i + (p_j - c_j)D_j. \tag{5.12}$$

Similarily, we can write the necessary first order optimality conditions

$$\begin{cases} \dfrac{\partial(\pi_i + \pi_j)}{\partial p_i} = 0 \\ \dfrac{\partial(\pi_i + \pi_j)}{\partial p_j} = 0. \end{cases} \tag{5.13}$$

The first one of these can be expanded like in the non-coalition case, except now we get an extra term for the utility of player j. Differentiating the equation yields

$$\frac{\partial \pi_i + \pi_j}{\partial p_i} = \frac{\partial}{\partial p_i}(p_i - c_i)D_i + \frac{\partial}{\partial p_i}(p_j - c_j)D_j \tag{5.14}$$

$$= D_i^0 \left(1 - \sum_{j=1}^{3} \frac{E_{i,j}\Delta p_j}{p_j^0}\right) + (p_i - c_i)D_i^0 \left(-\frac{E_{i,i}}{p_i^0}\right) + (p_j - c_j)D_j^0 \left(-\frac{E_{j,i}}{p_i^0}\right). \tag{5.15}$$

Now we can require that the partial derivative is zero, simplify the equation as in the non-coalitioner case, divide by D_i^0 and rearrange the terms. This yields a similar vector b and matrix M, but with extra terms:

$$b_i' = b_i + \frac{D_j^0 E_{j,i} c_j}{D_i^0 p_i^0} \tag{5.16}$$

and

$$\begin{aligned} M_{i,j}' &= M_{i,j} + \frac{D_j^0 E_{j,i} p_j^0}{D_i^0 p_i^0} \\ M_{j,i}' &= M_{j,i} + \frac{D_i^0 E_{i,j} p_i^0}{D_j^0 p_j^0}. \end{aligned} \tag{5.17}$$

Again, the Nash equilibrium can be solved by (5.11), but using M' and b' instead of M and b.

5.3.3 Conditions for the grand coalition

With all the players forming a coalition, every player tries only to maximize the total utility of all the players. The coalition faces the maximization problem

$$\max_{p_1,p_2,p_3} \pi_1 + \pi_2 + \pi_3 = (p_1 - c_1)D_1 + (p_2 - c_2)D_2 + (p_3 - c_3)D_3. \tag{5.18}$$

In a similar way, we can again derive the first order optimality conditions. The results are similar to the ones in (5.16) and (5.17), but now each player's conditions have a term for both of the other coalitioners. This ends us up with the equations

$$b_i'' = b_i + \frac{D_j^0 E_{j,i} c_j}{D_i^0 p_i^0} + \frac{D_k^0 E_{k,i} c_k}{D_i^0 p_i^0} \tag{5.19}$$

and

$$M_{i,j}'' = M_{i,j} + \frac{D_j^0 E_{j,i} p_j^0}{D_i^0 p_i^0}. \quad \forall i, j = 1, 2, 3, i \neq j \tag{5.20}$$

Now the Nash equilibrium can be solved by (5.11), but using M'' and b'' instead of M and b.

5.3.4 Conditions for limited spectrum

One important aspect of cognitive radio is that it uses the spectrum more efficiently. To determine the benefit of this, we must calculate its effect on the sales volumes and prices. If the spectrum runs out, it will affect mainly the network operator, since they simply cannot sell any more subscriptions. Device and service sales will also diminish, but this will happen mostly because network subscription fees will be higher.

As before, each player maximized their profit by solving the problem (5.3), but now the network operator has an extra constraint, since it cannot sell more than L subscriptions. This can be formulated as

$$D_N = D_N^0 \left(1 - \sum_{j=1}^{3} \frac{E_{N,j}\Delta p_j}{p_j^0}\right) \leq L, \tag{5.21}$$

where N is the index for the network operator.

The Karush-Kuhn-Tucker [4] conditions for this problem are

$$\begin{cases} \nabla \mathcal{L} = \nabla\left[(p_N - c_N)D_N + \lambda(D_N - L)\right] & = 0 \quad (5.22) \\ \lambda(D_N - L) & = 0 \quad (5.23) \\ \lambda & \geq 0. \quad (5.24) \end{cases}$$

If we assume that the solution satisfies (5.21), the constraint is not active and therefore $\lambda = 0$. This yields the same optimality conditions as the original unconstrained problem. On the other hand, if the constraint is violated, this cannot be the optimum and the constraint must become active. Then λ must be positive and the Equation (5.23) requires that $D_N - L = 0$, i.e. $D_N = L$. Since there is only one variable and one equation, solving this gives the optimum.

In matrix form this means that if the optimal solution violates the constraint, we substitute the Nth row in matrix M with the multipliers from the constraint $D_N = L$:

$$M_{N,1-3} = \left[\begin{array}{ccc} \frac{E_{N,1}}{p_1^0} & \frac{E_{N,2}}{p_2^0} & \frac{E_{N,3}}{p_3^0} \end{array}\right], \tag{5.25}$$

and the Nth element in vector b with

$$b_N = 1 - \frac{L}{D_N^0} + \sum_{j=1}^{3} E_{N,j}. \tag{5.26}$$

5.4 Mathematical analysis

Next, we will discuss some properties of the model visible directly from the mathematical form. The model requires several input parameters, and estimating all of these parameters includes high uncertainties. It is therefore important to detect which parameters have the the largest influence on the end results and which have the least. Then we can take this into account while estimating parameters or analyzing the results.

5.4.1 Effect of fixed costs

Fixed costs were left out of the model, because they do not have any effect on the equilibrium. While differentiating in Equation (5.5), the constant fixed costs term would disappear. Thus, the fixed costs do not have any effect on the equilibrium points. This means that only the marginal costs affect the equilibrium. Of course, the fixed costs lower the absolute level of each firms utility, but always with the same amount. Another important factor is, that we are only interested in the changes brought in by cognitive radio, so the absolute levels do not matter. Instead we only need to consider the changes in the fixed costs.

On the other hand, it is very difficult to estimate the fixed costs. After evaluating the profits for each firm with and without cognitive radio, we can estimate the change brought to the ecosystem. If we set all fixed costs to zero, this change can now be interpreted as the maximum investments allowed, to keep cognitive radio still profitable.

5.4.2 Changes in demand parameters

The demand function could have many parameters, but essentially it can be simplified as the absolute level and the slope of the function. Therefore we can limit the changes in demand to two factors, uniform growth and constant growth. These different types of growth are depicted in Figure 5.1. These types of changes are especially important when analyzing the difference between different business scenarios.

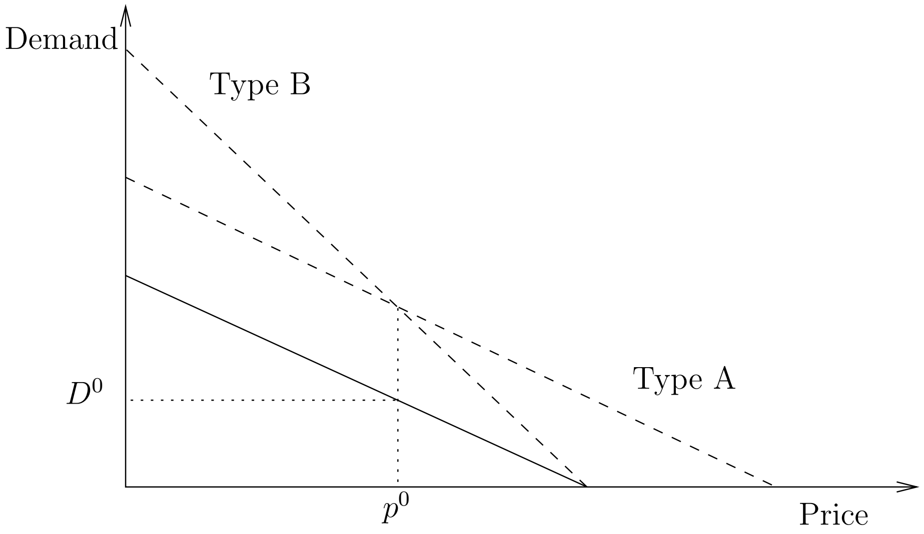


Figure 5.1: Demand change types

If the overall demand factor changes by a constant rate, in the mathematical sense this means that in the model the parameter D_i^0 changes. It is notable that this does not change the equilibrium point for the basic situation. In the coalition equilibria, the ratio of the changes between the different coalitioners matter. This means that to maintain the same equilibrium point, both initial demands must change at the same rate.

For the uniform percentual growth rate r, the demand function would be

$$D_i = rD_i^0 \left(1 - \sum_{j=1}^{3} \frac{E_{i,j} \Delta p_i}{p_i^0} \right). \tag{5.27}$$

Even though the change does not affect the equilibrium, it will still change the absolute level of the utility functions. If the equilibrium stays constant, the utilities will be scaled with the same factor as the demand functions. It is also noteworthy that if the changes are small compared to the elasticities, the equilibrium point will not move far from the original. Instead, the main effect from a uniform growth causes the same percentual rise in the utilities.

The constant growth factor on the other hand is a bit more complex. It means there will be a constant boost in sales, no matter what the price. This can be interpreted as new service types that attract new customers, but the elasticity of the new customers is still the same as for the older services.

Mathematically this means that there will be a constant added to the demand. If the percentual growth compared to the initial demand is r, we can write the demand functions as

$$D_i = D_i^0 \left(1 - \sum_{j=1}^{3} \frac{E_{i,j} \Delta p_i}{p_i^0} \right) + r D_i^0 = D_i^0 \left(1 + r - \sum_{j=1}^{3} \frac{E_{i,j} \Delta p_i}{p_i^0} \right). \tag{5.28}$$

Unlike the uniform growth factor, this will change the equilibrium. Following the calculations for the basic situation, we will get the b-vector for the new situation:

$$b = \begin{bmatrix} 1 + r_1 + \sum_{j=1}^{3} E_{1,j} + \frac{E_{1,1} c_1}{p_1^0} \\ 1 + r_2 + \sum_{j=1}^{3} E_{2,j} + \frac{E_{2,2} c_2}{p_2^0} \\ 1 + r_3 + \sum_{j=1}^{3} E_{3,j} + \frac{E_{3,3} c_3}{p_3^0} \end{bmatrix}. \tag{5.29}$$

However, the matrix M is not affected. Note that typically elasticities are usually greater than 1, which means the relative size of each r_i is small. This means that the equilibrium will again not change dramatically, but the utilities will get a constant boost proportional to the gross margin $p - c$.

5.4.3 Sensitivity

Most of the parameters are very difficult to estimate since first of all, there is a lot of uncertainty in the parameters and secondly there is almost no data available to base the estimates on. Therefore it is important to determine how much an error in a certain parameter affects the equilibrium prices and profits.

Evaluating the prices involves calculating the inverse 3-by-3 matrix M, which makes the analytical sensitivity estimates very hard to determine. For a numerical sensitivity analysis, we can easily compute the utilities for sets of parameters and numerically estimate the sensitivities.

Based on the Equations (5.8) - (5.11) of the vector b and matrix M, it easy to see that the largest effect on the equilibrium prices will be caused by changes in elasticity parameters, especially each players elasticity for their own price.

The initial price vector p_0 sets the scale for each price and cost, but does not affect the equilibrium in a qualitative sense. Prices and costs can be considered as a percentage of the initial price, as determined by Equation 5.11. The costs appear only in the b vector, and its magnitude is very small compared to the other terms in the vector. Thus the costs and initial prices do not have that much of an effect on the equilibrium.

Even though the equilibrium is not affected much, the overall utilities depend highly on the costs and price parameters. Changes in these parameters are directly visible in the utilities, and the changes in the parameters are proportional to the value of demand.

Although the equilibrium is not as sensitive to parameter changes as the overall utilities, it is possible that cognitive radio will shift the parameters dramatically. Then, even with small sensitivity, the end results will change notably.

5.5 Scenarios

As discussed before, cognitive radio is not yet fully defined. Therefore, because of the high uncertainty it is difficult to estimate the possible effects. To avoid this, we will construct different scenarios that represent some extreme alternatives. In addition to this we will look at a few more specific scenarios. Each scenario will imply a set of parameters, which we can then apply in the model.

As an overview to the scenarios, refer to the Figure 5.2.

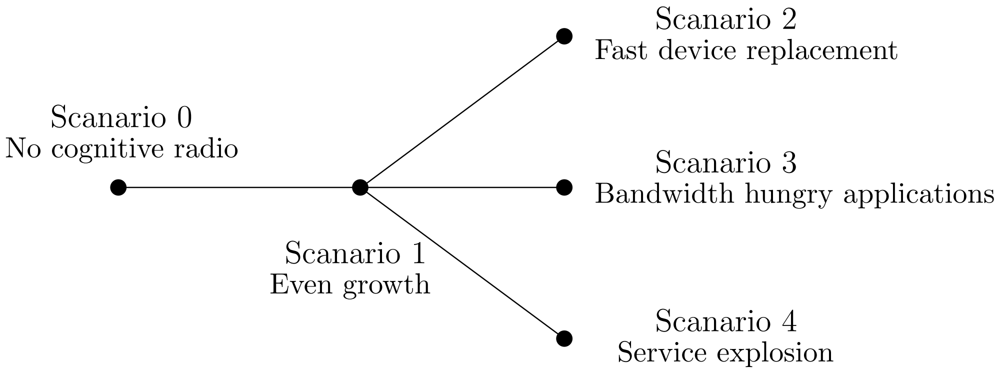


Figure 5.2: Overview of scenarios

5.5.1 Scenario 0: No cognitive radio

In the first scenario, we examine the situation where there is no cognitive radio. Device manufacturers sell ordinary terminals without cognitive capabilities and the operators sell data plans for use in their own network. Each operator buys licensed spectrum at governmental auctions, and host their own connection services in these bands. Whitespaces or other bands outside the ordinary mobile bands are not used for consumer markets. Base station networks are maintained by the operators and these networks provide all data access.

The data rate, coverage, capacity and other properties of mobile networks will stay at a normal level. Due to the growing popularity of mobile traffic, this will result in very high utilization rate of the current bands. In large cities, the spectrum will eventually run out, resulting in more demand than can be satisfied. This will result in higher network access prices.

5.5.2 Scenario 1: Even growth with cognitive radio

Cognitive radio has come to the consumer markets. The use of the new technology has made possible the introduction of new services that attract new customers. The new available spectrum has improved the network connections, making the mobile applications more pleasant to use. Also, old service subscribers use the services more since the quantity and quality of the services have grown along the increasing amounts of users. To access these services, consumers subscribe for data plans more actively, and buy new devices that can access these new services.

As a result, overall market demand levels grow. Although, the market growth affects all sectors equally, and none of the key players gain a clear advantage over the others.

5.5.3 Scenario 2: Fast device replacement

Cognitive radio has evolved in a similar way as in scenario 1, providing new and better quality services that increase the sales on every market sector. Cognitive radio has not yet evolved into its full form, and new spectrum bands are made available for cognitive use annually.

The new spectrum bands are used in parallel with the older bands, and controlled by the network operators. To fully utilize the potential of the available spectrum, users will have to upgrade their devices frequently. This has led to dramatically increased device sales. Although the other market sectors gain a clear increase in sales, the device manufacturer's increase is significantly larger.

5.5.4 Scenario 3: Bandwidth hungry applications

As in scenario 1 and 2, cognitive radio has made its way to the consumer markets, although spectrum allocation has reached a more mature stage. Consumers replace their devices at the same rate as before cognitive radio. New, better quality spectrum has improved the overall usage of mobile technology and thus increased the overall user amounts.

High bandwidth applications, such as video streaming and calls, have grown popular. More and more people augment their mobile phones with data capabilities to make use of these new applications. This has increased the need for extra bandwidth, and the network operator is obliged to build more capacity.

This has led to a situation, where the demand for network subscriptions has grown significantly more than the demand for the other products. Other sectors, especially the service sector, have not yet found a way to fully utilize the potential of these auxiliary services.

5.5.5 Scenario 4: Service explosion

Business has grown as in the previous scenarios. As in scenario 3, spectrum allocation is at a mature stage, and the new technology has enabled new, innovative services and improved the old services by increasing the network connection quality.

Unlike scenario 3, the service sector has managed to fully take advantage of the new applications. The increased number of subscriptions has enabled the services to improve further, attracting even more consumers. The application protocols have also been updated so that they require less bandwidth. To generate more profit, video calls and other high bandwidth applications are being charged more, which reduces the amount of network subscribers.

In general, the overall service subscription rates have grown dramatically, while the other sectors receive a more moderate increase in sales. Although, cognitive radio and its new features ensure growth on all sectors of the mobile telecommunications market.

5.6 Parameter estimation

5.6.1 Estimating initial prices and demand

We fix the time period in our model to one month. Network subscriptions are usually sold per megabyte or with a flat monthly fee. If we assume the per megabyte usage to be average, we can calculate the average monthly fee or the price of a data plan as a price for the network operator. Devices on the other hand are sold for a much longer period of time. We will scale the prices to match the equivalent time period as a network operator by dividing the price of an average phone with the estimated average lifespan of a smartphone, 24 months. For services the prices may vary a lot. This we can model with a single 1 euro service, but whose demand can be approximated be ten fold the demand of a network subscription. This ratio will then be the amount of money an average person will use on mobile services each month.

5.6.2 Estimating elasticities

Estimating the elasticities of demand is not a straightforward task. The uncertainty of the elasticities is very high, and data collection is hard to organize due to the need of large sample sizes. The elasticity also highly depends on the current level of prices. At a higher price, the elasticity can be different than at a lower price. Here we assume the elasticities to be independent of the price level.

The main methods for analyzing elasticities are market surveys and statistics. Under different prices, the demand is estimated, and then after enough data the changes are decomposed to effects caused by the general market status, prices and competitor prices. Typical tools include linear regression, time series analysis and other statistical methods.

The main problem is the amount of data needed. Data needs to be collected during a long period of time since prices changes only in long intervals, and sudden changes usually cause overreactions. Also identifying the possible other sources, such as branding [40], is important, as they might have a much greater effect on the demand, than the prices do. While examining the market at a business sector level, it is also important to notice that the elasticities might be different to the wholesaler and retailer [42].

5.6.3 Estimating changes in demand parameters

The main aim of this study is to determine how cognitive radio affects the market. We focus on the market balance, so especially we need to determine the changes to the parameters that define it.

In the consumers point-of-view, the only visible change is that mobile devices have new properties which make them more interesting and desired. The manufacturers and operators on the other hand have to invest in research and development and possibly include new parts or services. In the business sense this only means more marginal and fixed costs.

To simplify the situation, let us only consider the changes in the demand and marginal costs. Fixed costs are only constant terms subtracted from the profits, so they can be analyzed later on. All the other parameters in the model map to the demand functions, so now we will look at only these three parameters.

Changes in the demand function can be mapped in several ways, but as we use a linear demand function, two parameters are sufficient to determine the whole function. Let us denote the *change in population* as the type A growth in Figure 5.1. This means that no one will pay more for the product, but some new property will attract more people to use it. As an example, new network capacity can enable more customers to use their e-mail accounts, but since the service is still e-mail, they will not want to pay more for it.

The other changes can be mapped to *change in trend*, type B growth. This means that the market gets new customers that will buy products, even with a higher price. This is typical for new technological advancements or features, which add value to the product, and for which the customers are willing to pay more.

Mathematically, population growth can be denoted as *uniform growth* and trendy growth as *constant factor growth*, as described in Section 5.4.2.

5.6.4 Parameter estimation network for cognitive radio induced changes

Next we need to map the effects of cognitive radio to the cost parameter and two demand parameters. To do this we will construct a parameter estimation network to identify the influences of each property of cognitive radio.

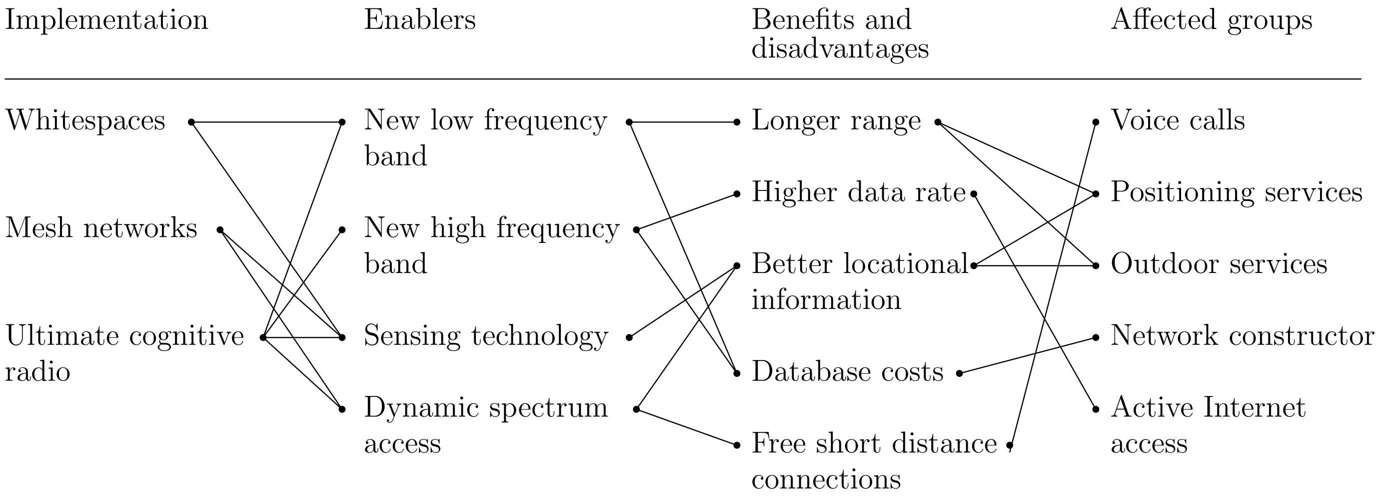


Figure 5.3: Parameter estimation network

The idea of the estimation model is illustrated in Figure 5.3. In the first column we have the different scenarios of implementations we wish to study. The second column denotes the cognitive radio enablers, all things that cognitive radio brings with it. Items in the two columns are connected if the scenario in the first column allows the new property. Note that these lists are not complete, the items only represent examples of each column.

The third column lists all the new benefits and disadvantages caused by cognitive radio. They are again connected with the second column, if the item in the second column induces the benefit in the third column. The fourth column represents the groups and users that benefit from cognitive radio. All items in the fourth column map to the players in the model. For example, local advertisers can be a part of the services sector by a factor of 30 %. The items in the third and fourth column are connected, if enabling the benefit from the third column affects the groups in column four. These connections also include information on how much the benefit effects the respective group.

As an example, let us examine the "Whitespaces" implementation in Figure 5.3, which includes the utilization of US TV whitespaces. Using whitespaces makes new low frequency bandwidth available and requires development of sensing technology, and hence they are connected with lines. New low frequency bandwidth as such does not benefit the end users directly, but it enables longer range connections. The lower frequencies are still licensed to the TV broadcasters, and FCC rules for utilizing the whitespaces dictate, that in order to use the band, the user must query a database to check whether the frequency is used by the licensed user.

By enabling a longer range, positioning services can improve their reliability and operating area. This attracts more customers, resulting in a 20 % growth in demand. Positioning services are used by 10 % of the total population, so in total there is a $20\ \% \cdot 10\ \% = 2\ \%$ growth in services. On the other hand, the database was needed to enable the use of the low frequency bands. The operator of the database can be considered as a part of the network operator business sector. The constructor of the network must also build the database infrastructure. The cost of the database can be estimated to 1 euro per user. Repeating the same logic to all connections in the network, we can sum up the total influence of cognitive radio.

Using this method to evaluate the changes of the market ecosystem makes it easier to estimate the actual parameters for the model. With this tool, it is possible to identify the sources of different changes, and the process can be done in a more systematic way. Although, even with this type of tool, predicting the future still has much uncertainty. It is not only difficult to estimate the actual effect of each benefit or disadvantage, but also it is impossible to know, what new types of benefits cognitive radio will bring with it.

Chapter 6

Numerical results

6.1 Parameter estimates

The estimation of parameters for cognitive radio is very difficult, since by now there is no cognitive radio available. Instead, we can estimate the parameters from traditional mobile communications, since they represent approximately the same area.

Our estimates are based on a scenario of the United Kingdom in the year 2015. By this point, the first cognitive radio devices have been built and the mobile communications market has matured from the current situation. The British telecommunications authority, Ofcom, also has a good selection of market data available at their website.

6.1.1 Initial demand and prices

To estimate the equilibrium prices for different products, we can study time series from various sources. The general trend seems to be that prices are going down and demand up. Smartphone prices could level down to 88 € and according to Ofcom, a typical mobile data plan will cost 10 € per month [34]. A typical lifespan of a smart phone could be 2 years, which gives us the average monthly price 88 €/24 = 3.67 €.

We estimate that an average consumer will use 10 € on other services, such as downloadable content or website subscription fees. But this total is not paid in one monthly payment, instead it is distributed between several smaller transactions. Let us assume the average service price is 1 €, and a typical consumer buys 10 of these every month.

The amount of mobile services consumers will rise. By 2015, approximately 50 million people could have a smartphone in the UK and half of these could have a data plan. This is because some people buy a modern smart phone, but do not actually intend to use the advanced features. We assume services to be sold for 10 € a month, for 1 € each. This sets the amount of services sold to be 10 times the amount of potential service buyers, by which we here mean data plan subscribers.

There are many other means of payment for data plans and such, but here we only consider the monthly flat fee payments. Payment methods based on usage can be averaged out as monthly fees with average usage data.

To summarize, the initial prices and demands are

$$p^0 = \begin{bmatrix} 3.67 \\ 10 \\ 1 \end{bmatrix} \quad \text{and} \quad D^0 = \begin{bmatrix} 50 \\ 25 \\ 250 \end{bmatrix}. \tag{6.1}$$

6.2 Elasticities

Estimating elasticities is very difficult due to the lack of sufficient data. Several studies by Perspective, Rappoport and Sidak [33, 39, 43] report the elasticity of demand for home broadband connections to be approximately 1.5-2 in the year 2009 with the current average price. In 2015 the elasticity will most likely be somewhat lower, but on the other hand mobile devices have slightly higher elasticities, since they are not still as necessary as conventional landlines. Therefore we will use the value 2 for the own-price elasticity of the network operator demand.

The elasticity for mobile devices is lower than for data plans. They are a necessity in the modern society, and data plans are, even though growing more popular, not obligatory. Therefore we will use the value of 1.5 for the device manufacturer's elasticity. Services are even more easier to give up if they are too expensive. They are not a necessity in any way, at least to some point. The elasticity can be set to 3.

In addition to the elasticities against the players own price, also the elasticities for the other players' prices are important. These elasticities should be lower than the elasticities for the players' own prices, since naturally the price does not directly affect the consumer.

Consumers usually first buy a mobile phone, then the data plan and finally services. The device and data plans are long lasting, and the current service prices will not affect the consumers choice. This why the cross-elasticities against the service prices are relatively low. We estimate them to be 0.5 for the device and 1 for the data plan. For the data plan it is slightly higher, since a consumer may think he will not take the subscription is the available services are too expensive, but still he will buy the phone.

Elasticities against data plan prices are higher than the ones against service prices. If the data plan is too expensive, it will make buying a smartphone seem useless and on the other hand the total spending on mobile communications will be too high and people expect to get more value for the plain data plan. We will use the values 1.0 for devices and 1.5 for services.

Changes in the price of the mobile device affect the whole business, since if there are less devices on the market, there is no capability to use the services. This is why the elasticities are relatively high, 1.5 for network and 1.0 for services demand.

Combining all this, we get an *elasticity matrix*. The element on row i and column j stands for the elasticity of demand for the ith player against the price of the jth player's price. Indices 1 refer to the device manufacturer, 2 to the network operator and 3 to the service provider. The elasticity matrix used is

$$E = \begin{bmatrix} 1.5 & 1.0 & 0.5 \\ 1.5 & 2.0 & 1.0 \\ 1.0 & 1.5 & 3.0 \end{bmatrix}. \tag{6.2}$$

6.3 Marginal costs

The costs for making a smartphone could be around 75% of the retail price, which corresponds to around 66 € over two years. Typically electronics marginal costs are quite low, and due to the need for research and development, initial fixed costs rise very high. Production volumes on the other hand are also high, so the fixed costs per product are also very low.

On the network and services sector the situation is not as easy. One could easily imagine, that the main source of costs is fixed costs. This is not fully true, since both operators and service providers need to construct the network or server infrastructure so that they are capable of handling the demand. At a small scale, there might not be any marginal cost for adding one consumer, but in a larger scale, most costs are marginal costs.

There is high uncertainty related to estimating the marginal costs. It is especially difficult to estimate the overall marginal costs of a whole business sector. Although, the aim of this study is not to estimate the profitability levels of each business sector, but evaluate the changes by cognitive radio. This means that more important than the level of marginal costs is how they change.
A summary of the parameters in marginal costs is listed in Table 6.1.

	Marginal costs
Device	66 € (over 2 years)
Network	6 €
Service	0.6 €

Table 6.1: Summary of marginal costs used in the model

6.4 Changes from cognitive radio

The most important parameters in the model are the changes brought by cognitive radio. As discussed in Section 5.6.3, we will limit the changes to only 3 parameters.

Ofcom reports an annual growth of 5-10 % for mobile subscriptions around the year 2008 [34]. Because we believe that cognitive radio will in any case bring some benefits, we expect at least the same amount of extra growth from cognitive radio. For the first scenario, we will assume all three sectors gain +10% growth. This will we be evenly distributed as population and trendy growth.

In the three other scenarios, Scenarios 2, 3 and 4, one of the players had an advantage over the others. The player with the advantage experiences an extremely high growth in demand. This can be compared to the beginning of the 21st century, when the mobile telecommunications era was starting. At that time Ofcom reports growth factors of 15-35 %. We will assume a growth of +20% for the advantaged player, and the growth will be evenly distributed between population and trendy growth.

Also the marginals costs will most likely change, but it is relatively difficult to estimate how much. For the device manufacturer the addition of one new property is not very expensive, around 1 € per device. The main sources of costs will be fixed research and development costs. The network and services have a similar situation. For them the exact amount is even harder to predict, thus we will assume there is no extra costs for implementing cognitive radio. Since there is so much uncertainty in these numbers we will examine the importance of them in the sensitivity analysis section.
A summary of the parameter changes for Scenarios 1-4 is listed in Table 6.2.

	Changes in demand (Population/Trendy)				Changes in marginal costs
	Sceneario				
	1: Even growth	2: Fast device replacement	3: Bandwidth hungry applications	4: Service explosion	
Device	+5%/+5%	+10%/+10%	+5%/+5%	+5%/+5%	1 €
Network	+5%/+5%	+5%/+5%	+10%/+10%	+5%/+5%	0 €
Service	+5%/+5%	+5%/+5%	+5%/+5%	+10%/+10%	0 €

Table 6.2: Summary of model parameters

6.5 Results

Now we will examine the results given by the model. First we will discuss the overall profitability of investing in cognitive radio for each player. Then, we will study the case that the spectrum is limited without cognitive radio, and see how it affects the benefits for each player.

6.5.1 Overall model properties

Using the parameters in the previous sections and applying them to our model, the Nash equilibrium of the business ecosystem game for Scenario 0 (No cognitive radio) and 1 (Even growth) without coalitions is described in Table 6.3.

Scenario 0: No cognitive radio

	Price	Demand	Profit
Device	4.5	35.7	62.1
Network	9.8	19.1	73.0
Service	0.93	250.2	83.5

Scenario 1: Even growth

	Price	Demand	Profit
Device	4.6	38.1	67.8
Network	9.9	20.3	78.2
Service	0.94	265.8	89.7

Table 6.3: Nash equilibrium of Scenarios 0 and 1

By calculating the differences, we get the total changes induced by cognitive radio, listed in Table 6.4.

Changes

	Price	Demand	Profit
Device	0.08	2.5	5.6
Network	0.04	1.2	5.2
Service	0.004	15.6	6.3

Percentual changes

	Price	Demand	Profit
Device	1.7%	7.0%	9.0%
Network	0.4%	6.1%	7.1%
Service	0.4%	6.3%	7.5%

Table 6.4: Changes induced by cognitive radio

In Scenario 1, all demand parameters grew with a factor of 10%. The prices did not change dramatically, only by a slight increase. This means that the equilibria are very close to each other and thus the only change in the utilities is because of the increased demand and small changes in marginal costs.

Even though the demand parameters rose by 10 %, the actual change in total demand was only 6-7%. This is mostly because the prices rose slightly thus lowering the demand. Another small effect is because the equilibrium of Scenario 0 is not exactly the same as the initial price parameters p^0. To get the initial prices to match the equilibrium prices, the marginal costs should be adjusted. Because now we focus on the changes by cognitive radio, we will not go into detail adjusting these parameters as they do not effect the results significantly.

All three players gained the same 10 % factor growth in demand, but the device manufacturer gained 2% more profits than the network operator or service provider. This happened even though the device manufacturer was the only one with raised marginal costs. The reason is elasticity structure. Population growth does not affect the elasticities, but the trendy growth does. Effectively it lowers the elasticity slightly and allows the seller to raise the price without losing too much customers. The device manufacturer, who has the lowest elasticities, can more effectively utilize this effect, as they can push up the prices even more.

Next let us look at the results from Scenarios 2, 3 and 4. The percentual changes in prices, demands and profits are listed in Table 6.5.

Scenario 2: Fast device replacement

	Price	Demand	Profit
Device	3.2%	16.5%	23.5%
Network	-0.3%	4.2%	3.4%
Service	0.3%	5.8%	6.6%

Scenario 3: Bandwidth hungry applications

	Price	Demand	Profit
Device	1.3%	6.0%	7.0%
Network	1.9%	15.4%	21.1%
Service	0.1%	5.3%	5.6%

Scenario 4: Service explosion

	Price	Demand	Profit
Device	1.6%	6.8%	8.6%
Network	0.2%	5.5%	6.1%
Service	1.4%	14.3%	18.7%

Table 6.5: Changes induced by cognitive radio in Scenarios 2, 3 and 4

In each scenario one player gained a significant advantage over the others. The player with the advantage had a growth factor of 20%, whilst the others stayed at the 10% rate as in Scenario 1.

The player with the advantage gained, as could be expected, a clearly better profit than the other players. Surprisingly, even though the other two players' demand parameters grew by 10%, their profits only grew by factors as low as 3%. The advantaged players, on the other hand, gained slightly more profits than 20%. The risen demand allows the advantaged player to raise its prices and therefore lower the other players demands. The others, especially the network operator in Scenarios 2 and 4, cannot raise its prices. They might even have to lower their prices to prevent losing too much demand. As the others lower their prices, the advantaged player gain even more demand and thus profits.

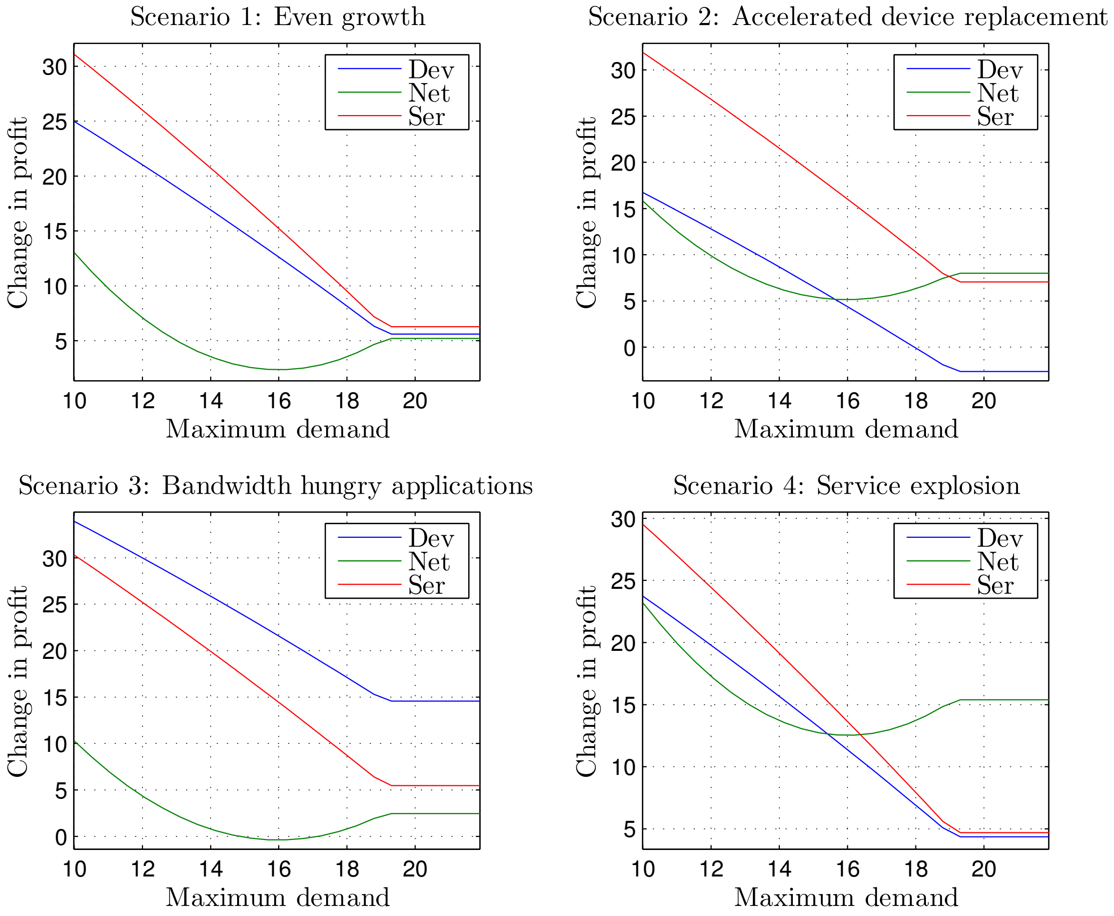
In general, the device manufacturer seems to be able to utilize its low elasticities to gain the highest profits. Although, we should still remember that in every scenario cognitive radio is profitable to all players.

6.5.2 Limited spectrum

One of the most important reasons cognitive radio is studied for, is the more efficient use of spectrum. Eventually the spectrum will run out, if nothing is done and the amount of traffic grows at its current pace. How much will the amount of needed spectrum grow depends on the future applications and number of users. If the traffic caused by a single user increases a lot, the bandwidth limit will be reached earlier and even if there are not as many users utilizing the spectrum. On the other hand, if applications will use less bandwidth and thus spectrum, more users will fit into the existing spectrum.

We will now study the effect of limited spectrum and how it affects the business ecosystem. We assume that services and device manufacturers are not directly affected, if the spectrum runs out. Instead, the network operator simply cannot sell more data plans than it is possible to fit in the available spectrum. This will naturally affect the device and service sectors indirectly. We will also assume that with cognitive radio, all of these limits are removed.

Now we can calculate the difference in profits when the spectrum is limited without cognitive radio and not limited with cognitive radio. Even though cognitive radio does not provide unlimited spectrum, the limit is so high we do not have to take it into account. The changes in profit if cognitive radio is implemented are presented in Figure 6.1.


Scenario 1: Even growth

Scenario 2: Accelerated device replacement

Scenario 3: Bandwidth hungry applications

Scenario 4: Service explosion

Figure 6.1: Effect of limited spectrum in different scenarios

Each figure shows a similar situation. When the maximum amount of demand is high, the players' profits are the same as in the previous results, since the limit is not active. When the maximum demand limit drops below 19, the limit becomes active and the profits start to change.

When the limit is between roughly 13 and 19, limited spectrum is actually beneficial to the network operator. This is because now the operator will raise its price until the demand limit is reached. Other players will have to adapt to this and lower their prices not to lose all their demand. This is why it is the device manufacturer and service provider, who get lower profits if the spectrum is initially limited. If the situation without cognitive radio is more beneficial to the network operator, the benefits of cognitive radio are obviously not as high as they would be without the limit.

When the limit drops below 13, cognitive radio will be more beneficial to all players compared to the situation without the limit. Then, if without cognitive radio, the network operator must raise its price so high that all players lose their demand and cannot lower their prices any more. This results in very low profits to all players, making cognitive radio relatively much better.

One important question is what is the true limit for the spectrum. At the current spectrum usage rate there is usually enough spectrum available, but in addition to the amount of subscribers, the amount of bandwidth and spectrum needed by each subscriber will grow in the future. It depends on the total effect of these two, how many subscribers will fit into the spectrum using current technology. If the limit is very low, cognitive radio will be very beneficial to all players. If the limit is slightly higher, cognitive radio might not be as profitable. Although, in the long run we estimate the amount of needed spectrum to grow, thus lowering the limit and making cognitive radio profitable.

6.5.3 Coalitions

So far all results have only considered situations, where the players do not cooperate. Now, we will look into coalitions, where two or three players decide their prices together. This way they can achieve higher profits in total, even though one of the players gets less. The results for all possible coalitions in Scenario 0 (No cognitive radio) are listed in the Table 6.6.

Coalition	No Coal	Dev./Net.	Dev./Ser.	Net./Ser.	Grand co.
Device	62.1	18.1	50.8	75.0	5.2
Network	73.0	135.0	100.0	60.8	145.9
Service	83.5	101.6	89.3	95.3	114.6
Coalitioners total	N/A	153.1	140.1	156.2	265.7
Non-coalitioners total	218.6	101.6	100.0	75.0	N/A
Benefit for coalitioners		+17.9	-5.5	-0.3	47.1

Table 6.6: Changes in coalitions profits

In general, coalitions should always benefit the coalitioners, since if they would gain less total profit while in the coalition, they could select the same prices as they would have chosen without the coalition. Now, this does not directly apply since we only consider the game's Nash equilibrium. The device/service and network/service coalitions actually produce less profit, than the coalitioners separately while not in the coalition. The reason is that now the third player also reacts to the prices set by the coalitioners. This effect could be changed by calculating the equilibrium as a two phase Stackelberg game, where first all the players set their prices separately and then the two players select their coalition prices. This setting does not affect the profits much, and therefore we will continue to use the Nash equilibrium in our analysis, since main aim is not to model the coalitions, but the effect of cognitive radio on these coalitions.

As can be seen from the table, usually when a coalition is formed, one player in the coalition gains most of the profit and the other significantly less. Most clearly this is visible in the device/network coalition. The reason for this type of behavior is that when the other lowers its price, the demand for both players' products rise. Then, the other player can rise its price and generate extra revenue from the newly grown demands. Note that this type of equilibrium requires that the profits are transferable.

The player, who will lower its price, depends on the elasticities and the amount of demand. For example, the network operators elasticity against the price set by the device manufacturer is relatively high. Now, if the device manufacturer sets a lower price, the effect in demand for the network operator is larger than the other way around. On the other hand, as the device manufacturer elasticity against the network operators price is low, it does not affect the demand that much, even though the network operator would ask a higher price for its services.

We can also calculate the Shapley values for each player. The Shapley value characterizes the importance of each player in all the possible coalitions. The sum of the Shapley values is the maximum total profit, which is the profit of the grand coalition. The Shapley values denote how big a share should each player get. The values for Scenario 0, No cognitive radio, are listed in Table 6.7. The Shapley values are very evenly distributed, which means that none of the players is more critical in forming coalitions than the others.

	Shapley value	Percent of total
Device	80.0	30 %
Network	93.5	35 %
Service	92.2	35 %

Table 6.7: Shapley values for Scenario 0: No cognitive radio

The important question is, how does cognitive radio change this setting? The Table 6.8 lists the changes in utility for each coalition.

In general, coalitions benefit from cognitive radio. In many cases, most of the increased demand benefits the players, but it does not affect the equilibrium qualitatively. Coalitions still have one player lowering and the other rising its price, which leads to the other player pumping up the demand, while the other collects profits. In Scenarios 2, 3 and 4, where one of the players has a advantage in demand growth, the equilibrium is shifted towards that player. It is now more beneficial to collect profits with the player that has clearly more demand. Although, the changes are yet so small that it does not change the overal situation the other way around. This means that for example in the device/network coalition, the device is still lowering its price, while the network collects most profits.

Scenario 1: Even growth					
Coalition	No Coal	Dev./Net.	Dev./Ser.	Net./Ser.	Grand co.
Device	9.0 %	15.2 %	9.3 %	8.9 %	37.7 %
Network	7.1 %	6.8 %	7.2 %	7.0 %	6.7 %
Service	7.5 %	7.6 %	7.5 %	7.5 %	7.6 %
Coalitioners total	N/A	7.8 %	8.2 %	7.3 %	7.7 %
Non-coalitioners total	7.8 %	7.6 %	7.2 %	8.9 %	N/A
Scenario 2: Fast device replacement					
Coalition	No Coal	Dev./Net.	Dev./Ser.	Net./Ser.	Grand co.
Device	23.5 %	115.3 %	27.9 %	22.4 %	476.1 %
Network	3.4 %	-4.4 %	2.5 %	2.6 %	-7.3 %
Service	6.6 %	6.5 %	5.8 %	6.8 %	6.8 %
Coalitioners total	N/A	9.7 %	13.8 %	5.1 %	8.3 %
Non-coalitioners total	10.3 %	6.5 %	2.5 %	22.4 %	N/A
Scenario 3: Bandwidth hungry applications					
Coalition	No Coal	Dev./Net.	Dev./Ser.	Net./Ser.	Grand co.
Device	7.0 %	-73.6 %	7.3 %	5.9 %	-371.4 %
Network	21.1 %	29.7 %	19.5 %	27.0 %	34.2 %
Service	5.6 %	6.8 %	5.6 %	3.5 %	3.8 %
Coalitioners total	N/A	17.5 %	6.2 %	12.6 %	13.1 %
Non-coalitioners total	11.2 %	6.8 %	19.5 %	5.9 %	N/A
Scenario 4: Service explosion					
Coalition	No Coal	Dev./Net.	Dev./Ser.	Net./Ser.	Grand co.
Device	8.6 %	17.1 %	5.5 %	10.2 %	27.4 %
Network	6.1 %	5.6 %	8.6 %	0.9 %	3.9 %
Service	18.7 %	18.1 %	19.4 %	20.8 %	21.0 %
Coalitioners total	N/A	7.0 %	14.3 %	13.0 %	11.7 %
Non-coalitioners total	11.6 %	18.1 %	8.6 %	10.2 %	N/A

Table 6.8: Changes of coalition profits

In almost every scenario, the coalition gains approximately 10 % extra profit, if the advantaged player is not part of it. On the other hand, if the advantaged player does take part in the coalition, the total gain is around 13-14 %. The player not taking part in the coalition usually gets similar gains as it would without the coalition game. On overall, coalitions do not effect the market balance dramatically. It does shift the balance inside the coalitions, but the effect outside the coalition is only small.

The Shapley values of each scenario are listed in Table 6.9. The values confirm similar behavior as the general changes in profits. In Scenario 1 the balance is not shifted in any way. In Scenarios 2-4 the players with the advantage gain a 10 % increase in their Shapley value. The interpretation of Shapley values suggests that in the case of the grand coalition, the Shapley values indicate how the profits should be divided. In Scenario 1, the players are fairly equal, and thus the profits should be equally distributed. In the other scenarios the advantaged player should get a slightly larger share.

Scenario	1: Even growth	2: Fast device replacement	3: Bandwidth hungry applications	4: Service explosion
Device	30 %	33 %	30 %	29 %
Network	35 %	33 %	38 %	34 %
Service	35 %	34 %	33 %	37 %

Table 6.9: Shapley values for each scenario as percentage of total

6.6 Sensitivity analysis

Using the model involves estimating several parameters which all have high uncertainties. It is important to detect the parameters that have a significant influence on the end results. This way possible error sources can be identified and prevented. First we will take a look at the effect of marginal costs and then study the effect of different demand growth types.

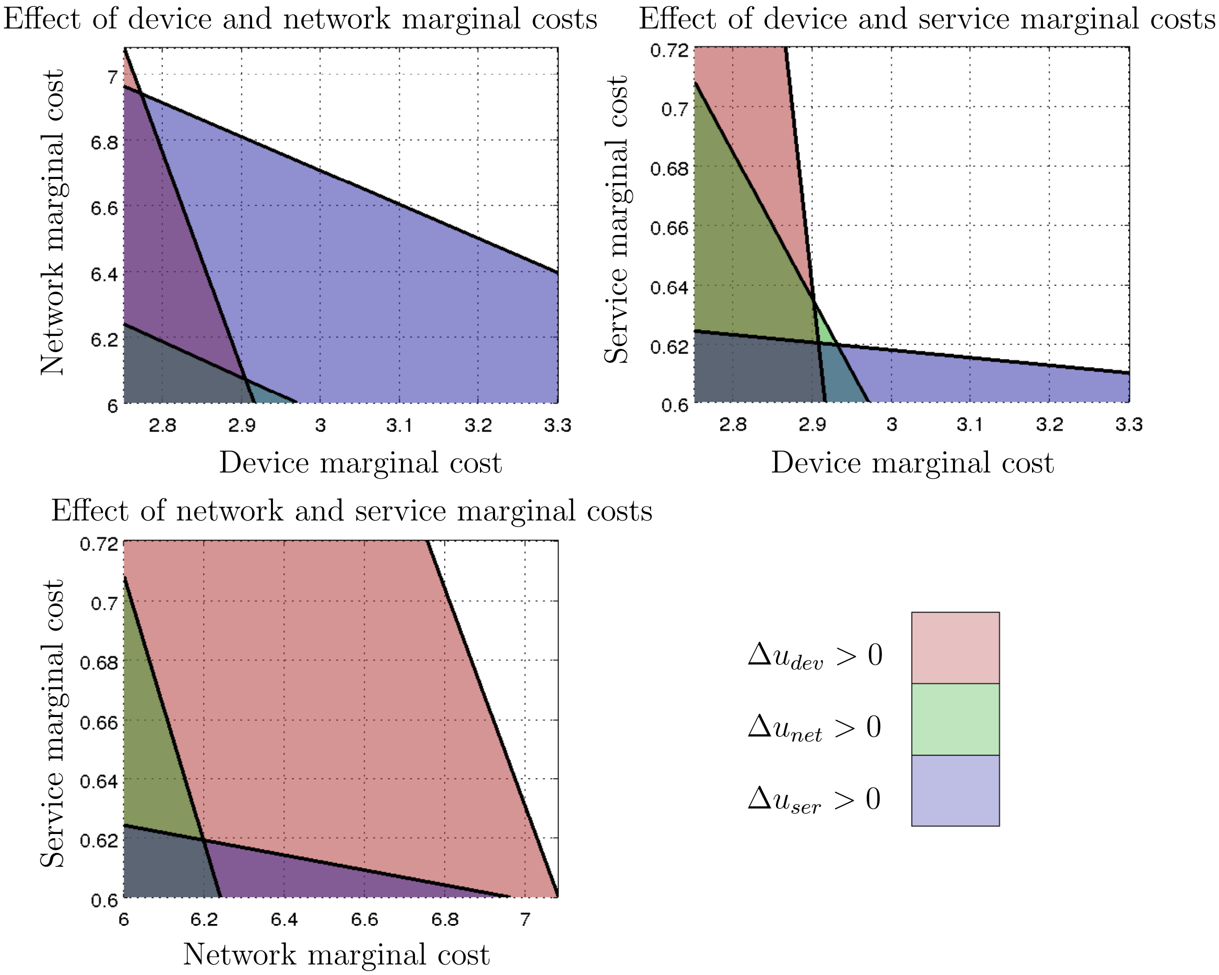
6.6.1 Marginal costs

We estimated that the device manufacturer's marginal costs will grow by 1 € per device. Network and service costs would not change significantly. This might not be true and for instance adding cognitive capabilities to base station network would increase the costs.

Figure 6.2 shows how much marginal costs can change so that cognitive radio will still be profitable to each player. Each area represents the possible increases for a single player, and the gray area in the bottom left corner indicates the area profitable to all players.

As could be expected, each player is most sensitive to his own marginal cost. Although, it is notable that the device manufacturer is not as sensitive to the other players' marginal cost changes, and on the other hand, the network operator is relatively sensitive to the device prices.

In general marginal costs may rise about 5 % for cognitive radio to still be profitable. Note that here profitable means that all marginal costs are covered, but fixed costs are not yet considered. This will shrink the areas by a factor, but as discussed in the previous chapter, it is very difficult to estimate these costs.



Effect of device and network marginal costs

Effect of device and service marginal costs

Effect of network and service marginal costs

Figure 6.2: Changes in marginal costs still profitable to each player in Scenario 1.

The changes of marginal costs for the device manufacturer were initially estimated to be around 1 € for one device, which divided by the 2 year life span of the device yields a 0.04 € monthly costs increase. This is roughly 1-2 % of the original marginal cost. Even with similar increases to the other players, cognitive radio would still be profitable to all players.

Next we will look at the sensitivity to marginal costs in Scenario 4: Service explosion. Similar graphs as with Scenario 1 are shown in Figure 6.3. Scenarios 2 and 3 give similar results, except with a different advantaged player.

The sensitivities are very similar to the ones in Scenario 1. Now only the advantaged player allows twice as much growth as before in all marginal costs until cognitive radio becomes non-profitable.

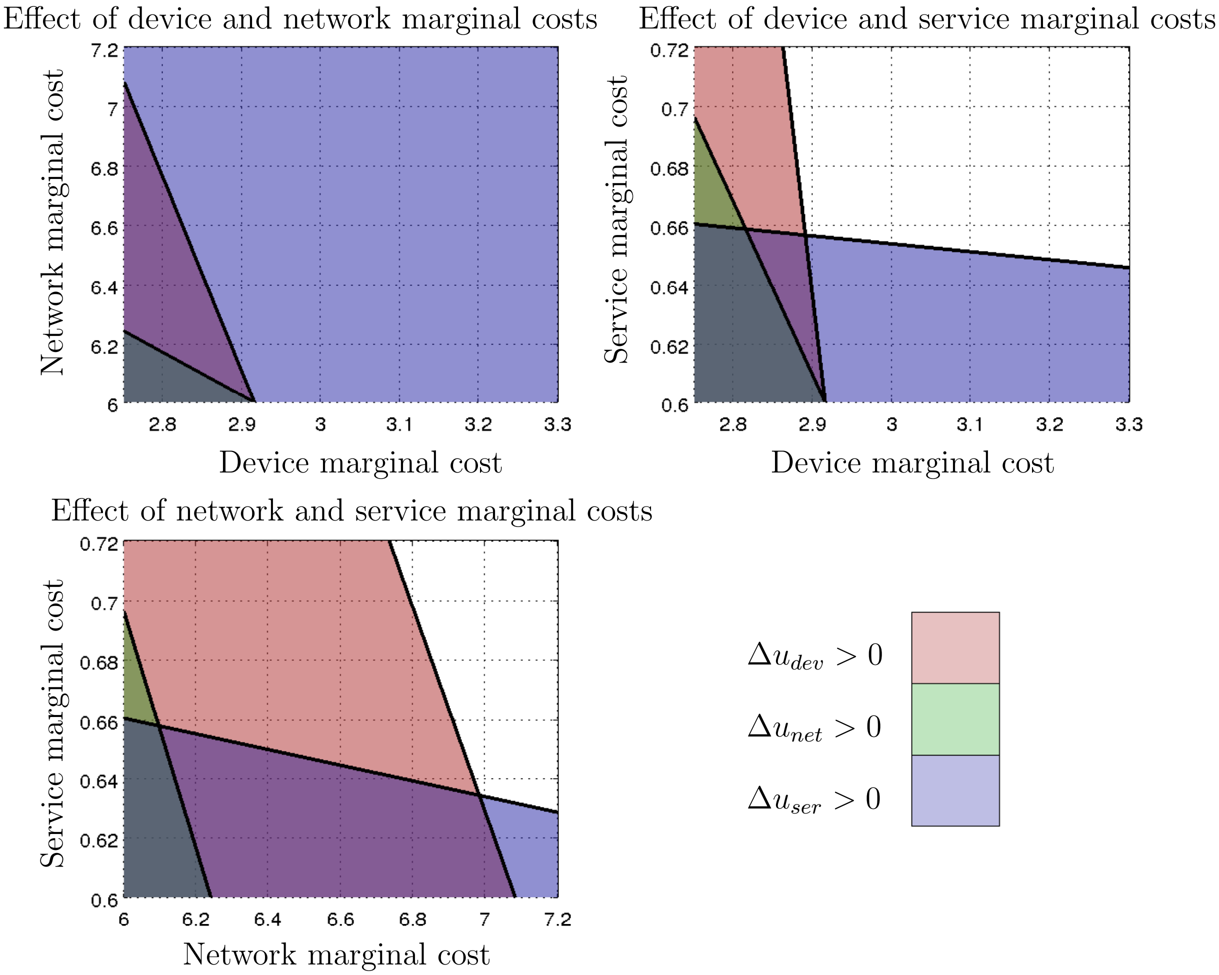


Figure 6.3: Changes in marginal costs still profitable to each player in Scenario 4.

In general, marginal costs may rise relatively much until cognitive radio is not profitable. This is, of course, subject to the other parameters. If the initial level of marginal costs is clearly higher, the overall profitability will not be as good. Then changes in marginal costs will also be less tolerated. For cognitive radio to be profitable, increases in fixed costs must also be covered. This means that although a high marginal cost is still profitable, it might not be enough to cover the fixed costs.

6.6.2 Trendy versus population growth

The growth in demand was divided into two different types of growth, trendy and population growth. In the basic scenarios we assumed an even ratio in both types. However, these two types of growth may change the equilibrium point significantly. In Table 6.10, the results of Scenario 2: Fast device replacement, have been recalculated. The non-advantaged players still have the growth factors of 5 % in both types, totaling to 10 % growth. The advantaged player, the device manufacturer, gains similar 20 % growth, but the equilibrium is calculated with different ratios of the two types.

Population/trendy growth	20 %/0 %	15 %/5 %	10 %/10 %	5 %/15 %	0 %/20 %
Device	14.9	19.4 %	23.5 %	27.0 %	30.0 %
Network	11.0	7.1 %	3.4 %	-0.3 %	-3.9 %
Service	8.5	7.5 %	6.6 %	5.6 %	4.7 %

Table 6.10: Changes of profits with different device demand growth types.

From the results we can clearly see that trendy growth is generally more profitable growth for the player receiving it. The device manufacturer's profit range from 15 % to 30 %, which means trendy growth is approximately twice as beneficial as population growth. From the other players' point-of-view, the situation is the opposite. The sum of all growth percentages is approximately constant, so if the device manufacturer gains less profits, the network operator and service provider gain more. It is also notable that network operator's sensitivity to the ratio of the two types of growth is higher than the service provider's.

The reason for this extreme behavior is that trendy growth changes the elasticities and therefore causes major shifts in the price equilibrium. In general trendy growth lowers the elasticity, which allows the player to raise prices without losing demand. The effect is enforced when other players must lower their prices to keep their current demand.

Trendy growth as denoted here seems to be slightly too powerful. A 10 % growth in trendy growth has a larger effect, since it changes the elasticity of every users, and does not just add a smaller population with a different elasticity. When estimating parameters, trendy growth should include a slight negative growth in population to reduce this effect.

Chapter 7

Conclusion

We believe that there definitely will be a cognitive radio in the near future. The only question is, when and in what form. Based on the results of this thesis, under the assumptions made for the United Kingdom in 2015, cognitive radio will satisfy the requirements of the mobile telecommunications market. Of course, it is currently impossible to predict how cognitive radio will be implemented in the end and how will the public audience receive it. With or without cognitive radio, the spectrum will run out if the use of mobile technology continues to grow at its current rate. Clearly, cognitive radio has several benefits consumers are willing to pay for, and thus we claim it is safe to assume that it will not be long until the first cognitive radio devices are available for the general public.

The model and the parameter estimates introduced in Chapter 6 suggest that the player to potentially receive the most benefits from the implementation of cognitive radio is the device manufacturer. The reason is mostly because of the low elasticities to all prices. Although, this is valid only under the assumption that every player gains equal growths in demand from the new technology. Whether this is the actual case depends highly on how cognitive radio will actually work. On the other hand, it is logical that it is the device manufacturers gaining the most benefits, as they currently put the most effort into researching the possibilities of cognitive radio. Although the model suggests that the device manufacturer will get the most benefits, one should still remember that cognitive radio will be beneficial to all the players.

One interesting result is the network operator's reaction to limited spectrum. In the short term, limited spectrum seems to be beneficial for the network operators. However, cognitive radio development takes some time and it will be beneficial also for the network operators in the long term. It should also be noted that in real life operators want to actively ensure that they are able meet the expected increase in demand. Cognitive radio will help operators to cope with the increasing traffic and enable new services for customers.

The model we constructed in this thesis provides a useful tool to evaluate the benefits of cognitive radio. There is still some work to be done, especially in selecting the parameters. So far the parameters used do not necessarily represent any exact situation, and with more market research the estimates of each parameter could be more accurate. The sensitivity analysis shows us that although the results given by the model are not qualitatively very sensitive to the changes in the parameters, quantitative changes can still be significant. A major factor in estimating the benefits is actually how the public receives the technology.

Although it can be considered as basic economic knowledge, the model reminds us of some of the basic principles of the business ecosystem. Each firm should find a way to generate demand with low elasticity. Brand loyal consumers ensure a steady income, but also allows the firm to use this loyalty to take away profits from the other players in the market. Typically this type of loyalty is very hard to generate, but new innovations related to cognitive radio could actually help create new products with devoted customers.

For a more in-depth analysis of a specific cognitive radio scenario, the thesis introduced the parameter estimation network for the model parameters. With this tool, different features of cognitive radio can be evaluated, since the estimation model helps find the connections between different technological properties and actual consumer preferences. This tool can be used as an aid for research portfolio decision making when deciding which features of cognitive radio are the most important ones.

For further research, an important step is to break down each of the three main players. Currently, they represent the whole business sector, not just a single firm. Modeling the sectors will definitely bring more depth into the results. In addition, some important factors, such as competition within a sector, have not been studied in this model so far. Competition could dramatically affect the way the parameter estimates are evaluated, or even influence the price setting capabilities.

There is still lot of work to do to make cognitive radio a part of the average users everyday life. The technology has to be developed, regulation has to be changed and there must be someone to buy the actual product and use it. New business models must be developed to sell the new technology. Cognitive radio might dramatically change the whole mobile business ecosystem and even alter the way we think of wireless communication. Also the concept of spectrum ownership will most likely become more dynamic. Spectrum might no longer be the privilege of network operators, instead it could become a commodity. This will give birth to new market players and make predicting the future market even more difficult. It might even turn out that the current representatives of each business sector do not have to benefit from cognitive radio or even exist anymore. They can be replaced, but by whom?

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